Pearson Edexcel

Mark Scheme (Results)

November 2020

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers normally score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

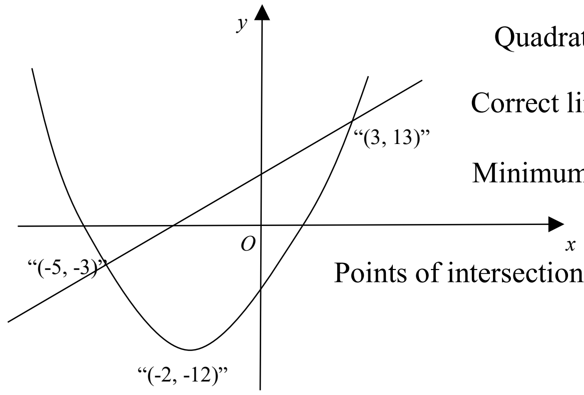
Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

International GCSE Further Pure Mathematics – Paper 1 mark scheme

Question number	Scheme	Marks
1 (a)	$t = \frac{10}{3}$ $P = 3 + 2\sin\frac{5\pi}{4}$ $= 3 - \sqrt{2}$ oe (e.g. $3 - \frac{2}{\sqrt{2}}$)	 M1 A1 (2)
(b) (i)	5	B1
(ii)	1	B1 (2)
(c)	$4 = 3 + 2\sin\left(\frac{3\pi t}{8}\right)$ $\frac{1}{2} = \sin\left(\frac{3\pi t}{8}\right)$ $\frac{\pi}{6} = \left(\frac{3\pi t}{8}\right)$ $t = \frac{4}{9}$ oe	 M1 M1 A1 (3)
		[7]

Part	Mark	Additional Guidance
(a)	M1	Correct substitution of $t = \frac{10}{3}$, leading to a value for P, simplification not required.
	A1	Answer stated, oe exact value.
(c)	M1	Correctly substitutes the value of $P = 4$ and rearranges to give an expression of the form $a = \sin\left(\frac{3\pi t}{8}\right)$ **Do not allow this mark if *a* > 1 or *a* < -1**
	M1	Correctly uses the inverse sin function to arrive at $b = \left(\frac{3\pi t}{8}\right)$ and solves to find a value of $\frac{3\pi t}{8}$, allow this value to be in degrees If the inverse sin function is not shown, then the value of the angle obtained must be correct for their *b*.
	A1	oe

Question number	Scheme	Marks
2 (a)	$\left(x+2\right)^2-4-8$ $\left(x+2\right)^2-12 \qquad a=2 \quad b=-12$	M1 A1 (2)
(b)	$x^2+4x-8=2x+7$ $x^2+2x-15=0$ $(x-3)(x+5)=0$ or any valid method $x=3, y=13 \qquad\qquad x=-5, y=-3$	M1 dM1 M1 A1 A1 (5)
(c)	 Quadratic drawn Correct line drawn Minimum labelled Points of intersection labelled	B1 B1 B1 ft B1 ft (4) **[11]**

Part	Mark	Additional Guidance
(a)	M1	Use general guidance, allow an expression of the form $\left(x\pm\frac{4}{2}\right)^2 \pm q \pm 8 \qquad q\neq 0$
	A1	Correct expression as shown, *a* and *b* need not be explicitly stated
(b)	M1	Correctly equates the 2 expressions
	dM1	Rearranges to a 3TQ = 0 (allow any 3TQ if intention of rearrangement is clear)
	M1	Uses any valid method to solve – **see general guidance**
	A1	For either pair of values stated
	A1	For all four values, correctly paired or written as coordinates.
	For the final A1 A1, do not allow recovery of *y* values from part c.	
(c)	B1	Correctly shaped quadratic curve, with a clear minimum point, drawn anywhere on their axis, mark intention.
	B1	Correct line – **must have a positive y intercept, a positive gradient and a negative x intercept**
	B1ft	Correctly labelled coordinates for their minimum, ft their answer from b, **must correctly ft their answer from a, ie minimum point labelled (- *a*, *b*)**
	B1ft	Correctly labelled coordinates for their intersections.
	The coordinates must be clearly indicated and not inferred from a scale on the graph. Ignore any labelling of intersections with axes.	

Question number	Scheme	Marks
3	$\ln 12 = \ln a + (2-1)\ln b$ oe	M1
	$ab = 12$ oe	A1
	$\ln 768 = \ln a + (5-1)\ln b$ oe	M1
	$ab^4 = 768$ oe	A1
	$\frac{768}{12} = \frac{ab^4}{ab}$ $(b^3 = 64)$	ddM1
	$b = 4$ $a = 3$	A1 A1
	ALT 1	
	$\ln a + (2-1)\ln b = \ln 12$ oe	M1 A1
	$\ln a + (5-1)\ln b = \ln 768$ oe	
	$3\ln b = \ln b^3$ $\ln 768 - \ln 12 = \ln 64$	M1 A1
	$b^3 = 64$	ddM1
	$b = 4$ $a = 3$	A1 A1
	ALT 2	
	$d = \ln 12 - \ln a$	M1
	$(d = \ln b = \ln 12 - \ln a \Rightarrow \ln b = \ln\left(\frac{12}{a}\right)) \Rightarrow b = \frac{12}{a}$	A1
	$\ln 768 = \ln a + \ln\left(\frac{12}{a}\right)^4$	M1
	$\ln 768 = \ln\left(\frac{12^4}{a^3}\right)$	A1
	$a^3 = \frac{20736}{768}$	ddM1
	$b = 4$ $a = 3$	A1 A1
	ALT 3	
	$(u_2 =)$ $u_1 + d = \ln 12$	M1 A1
	$(u_5 =)$ $u_1 + 4d = \ln 768$	
	$3d = \ln 768 - \ln 12$	M1
	$d = \ln 4$	A1
	$u_1 = \ln 12 - \ln 4 = \ln 3$ $(= \ln a)$	ddM1
	$b = 4$ $a = 3$	A1 A1 **[7]**

Part	Mark	Additional Guidance
	M1	Correct equation as shown oe
	A1	Correct equation as shown oe
	M1	Correct equation as shown oe
	A1	Correct equation as shown oe
	ddM1	**Dependent on both previous method marks**, uses any clear, valid method to reduce to an equation in a (or less likely, b)
	A1	For correct b
	A1	For correct a
ALT 1	M1	One correct equation as shown oe
	A1	Both correct equations as shown oe
	M1	Clear valid attempt to subtract one equation from the other
	A1	Achieves the two terms shown
	ddM1	**Dependent on both previous method marks,** uses a valid method to eliminate the logs and achieves an equation in b only
	A1	For correct b
	A1	For correct a
ALT 2	M1	Finds a correct equation as shown for the common difference, d
	A1	Correct equation as shown oe
	M1	Correct equation as shown oe (subs to get u_5)
	A1	Correct equation as shown oe
	ddM1	**Dependent on both previous method marks,** eliminates the logs and achieves an equation in a only
	A1	For correct b
	A1	For correct a
ALT 3	M1	One correct equation as shown oe
	A1	Both correct equations as shown oe
	M1	Clear attempt to subtract one equation from the other
	A1	Achieves the correct value for d in any single ln form
	ddM1	**Dependent on both previous method marks,** arrives at a single term log for u_1
	A1	For correct b
	A1	For correct a
	Allow full marks in general for just $b = 4$ and $a = 3$	

Question number	Scheme	Marks
4 (a)	$\frac{dy}{dx} = 3x^2 - 6x - 24$	M1
	$"(x+2)(x-4)" = 0$	M1
	$x = -2$ or $x = 4$	A1
	(–2, 34) and (4, –74)	A1 (4)
(b)	$\frac{d^2y}{dx^2} = "6x - 6"$ and substitution of their –2 or 4 or consider values of $\frac{dy}{dx}$ either side of their –2 or 4 or use properties of a cubic*	M1
	(–2, 34) maximum and (4, –74) minimum	A1cso (2) **[6]**

Part	Mark	Additional Guidance
(a)	M1	General guidance – an attempt to differentiate, power of at least one term must decrease by one. Also, no power must increase.
	M1	Equates their derivative = 0 and attempts to solve by any method **– see general guidance.**
	A1	Both correct values for *x*
	A1	All values correct, listed as coordinates or correctly paired $x = \ldots.. \; y = \ldots..$
(b)	M1	**Correctly differentiates their derivative** from part a and substitutes one of their *x* values. Allow sight of 18 or -18 following a correct differentiation to imply this mark.
	A1 cso	Allow this mark if there is an incorrect y value from part a Must correctly make the argument they've chosen and identify the points as shown. If substituting *x* = -2 and *x* = 4 or values either side of the these, their evaluations of the substitutions must be correct. * A convincing argument about the shape of a positive cubic curve and position of the minimum and maximum point must be made. Send to review, if in doubt. In the conclusion, it must be clearly stated which coordinate or value of *x* is a maximum and which is a minimum. Just a substitution of a value and stated max or min is insufficient.

Question number	Scheme	Marks
5 (a)	$1+\frac{1}{2}(-x)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(-x)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-x)^3$	M1
	$1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3$	A1 A1 (3)
(b)	$x = 0.08$	B1
	$1-\frac{1}{2}"(0.08)"-\frac{1}{8}"(0.08)"^2-\frac{1}{16}"(0.08)"^3$	M1
	0.959168 cao	A1 cao (3)
(c)	$(\sqrt{0.92}=\frac{\sqrt{23}}{5}\Rightarrow\sqrt{23}=)"0.959168"\times 5$	M1
	4.79584	A1 (2)
		[8]

Part	Mark	Additional Guidance
(a)	M1	For an attempt at a Binomial expansion. An attempt is defined as the following • The expansion must start with 1 • The powers of their $-x$ must be correct • $-x$ must be used at least once • The denominators 2! And 3! must be seen. Accept 2 and 6 Can be implied by at least 2 correct terms in an expansion
	A1	For at least one term in x correct and fully simplified.
	A1	For the expansion fully correct and simplified. Ignore terms in higher powers of x.
(b)	B1	For finding the value of $x = 0.08$
	M1	For correctly substituting **their** value of x into the expansion **provided** $\|x\| < 1$ Use of **their expansion** or the correct expansion must be seen explicitly here
	A1	cao
(c)	M1	For use of their value from (b) in $\sqrt{0.92}=\frac{\sqrt{23}}{5}\Rightarrow\sqrt{23}="0.959168"\times 5$
	A1	Cao
		Allow the final M1 A1 if $\sqrt{0.92}\times 5$ is clearly written and 4.79584 is clearly indicated as the answer.

Question number	Scheme	Marks
6 (a)	$(\sin A\cos B+\cos A\sin B)+(\sin A\cos B-\cos A\sin B)$ $=2\sin A\cos B$ *	M1 A1 cso (2)
(b)	$\sin 8x+\sin 6x$	B1 (1)
(c)	$3\int_0^{\frac{\pi}{4}}(\sin 8x+\sin 6x)\,\mathrm{d}x$	M1
	$=(3)\left[-\frac{1}{8}\cos 8x-\frac{1}{6}\cos 6x\right]_0^{\frac{\pi}{4}}$	A1
	$=(3)\left[\left(-\frac{1}{8}-0\right)-\left(-\frac{1}{8}-\frac{1}{6}\right)\right]$	M1
	$=\frac{1}{2}$ cao oe	A1cao (4) **[7]**

Part	Mark	Additional Guidance
(a)	M1	Correct expression show.
	A1	cso
(b)	B1	For the expression shown
(c)	M1	$k\int_0^{\frac{\pi}{4}}(\sin 8x+\sin 6x)\,\mathrm{d}x$ $k\neq 0$ or 1 k must be an integer. This mark can be awarded if the limits aren't seen on the integral.
	A1	Correctly integrated, the 3 and the limits need not be present for this mark
	M1	Correctly shown substitution of limits, with a subtract sign between. The 3 need not be present for this mark, need not be simplified. *This mark can be implied if first M1 A1 awarded and final correct answer.* Allow a correct substitution of limits into any changed expression.
	A1	cao oe

Question number	Scheme	Marks
7 (a)	$(V=)x^3$ $\quad (\frac{\mathrm{d}V}{\mathrm{d}x}=)3x^2$ $\quad$ (at $x = 2$ $\quad \frac{\mathrm{d}V}{\mathrm{d}x}=12$)	M1
	$\frac{\mathrm{d}x}{\mathrm{d}t}=0.1$	B1 (A1 on ePen)
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}=\text{"}12\text{"}\times 0.1$ oe	M1
	1.2 m^3/s cao oe	A1cao (4)
(b)	(Surface Area =) $6x^2$ $\quad (\frac{\mathrm{d}A}{\mathrm{d}x})=12x$ $\quad$ at $x = 6$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}x}=72$	A1
	$\frac{\mathrm{d}V}{\mathrm{d}x}=108$ $\quad \frac{\mathrm{d}A}{\mathrm{d}t}=0.05$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}=\text{"}108\text{"}\times\text{"}\frac{1}{72}\text{"}\times 0.05$	M1
	0.075 m^3/s	A1 (5)
	<u>ALT</u>	
	A = $6x^2$ $\quad$ leading to an expression in A for V $\quad V=\left(\frac{A}{6}\right)^{\frac{3}{2}}$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}A}=\frac{1}{4}\left(\frac{A}{6}\right)^{\frac{1}{2}}$ oe	A1
	$\frac{\mathrm{d}V}{\mathrm{d}A}=\frac{3}{2}$ and $\frac{\mathrm{d}A}{\mathrm{d}t}=0.05$ oe	B1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}=\text{"}\frac{3}{2}\text{"}\times 0.05$	M1
	1.2 m^3/s cao oe	A1cao [5] **[9]**

Part	Mark	Additional Guidance
(a)	M1	Correct expression for Volume, attempt at differentiation to ax^2, a is an integer, $a > 1$.
	B1	$\frac{dx}{dt} = 0.1$, can be explicit or implicitly used in a chain rule.
	M1	For **any correct chain rule**, that would lead to a value for $\frac{dV}{dt}$ and substitution of 0.1 and their value for $\frac{dV}{dx}$. They must show an attempt to find $\frac{dV}{dx}$, need not be a correct attempt, this isn't a dependent mark.
	A1	cao oe
(b)	M1	Correct expression for Surface Area, attempt at differentiation to bx, b is an integer, $b > 1$.
	A1	$\frac{dA}{dx} = 72$
	B1	$\frac{dV}{dx} = 108$ & $\frac{dA}{dt} = 0.05$ clearly stated, implicitly in chain rule or explicitly
	M1	For **any correct chain rule**, that would lead to a value for $\frac{dV}{dt}$ and substitution of 0.05 and their values for $\frac{dA}{dx}$ and $\frac{dV}{dx}$ They must show an attempt to find $\frac{dV}{dx}$, need not be a correct attempt, this isn't a dependent mark.
	A1	cao oe
(b) ALT	M1	Correct expression for area, attempt to rearrange, expression for V in terms of A
	A1	oe
	B1	Both derivatives clearly stated, implicitly in a chain rule or explicitly
	M1	For **any correct chain rule**, that would lead to a value for $\frac{dV}{dt}$ and substitution of 0.05 and their value for $\frac{dV}{dA}$. $\frac{dV}{dA}$ doesn't need to come from correct working, but there must have been some attempt to find an expression for V in terms of A and $\frac{dV}{dA}$ presented somewhere in the working.
	A1	cao oe

Question number	Scheme	Marks
8 (a)	$\alpha+\beta=\frac{1}{3} \qquad \alpha\beta=\frac{4}{3}$	B1
	$p=\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$	M1
	$=\frac{1}{3}+\frac{\frac{1}{3}}{\frac{4}{3}}=\frac{7}{12}$ *	A1 cso* (3)
(b)	$q=\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{1}{\alpha\beta}+\frac{\beta^2+\alpha^2}{\alpha\beta}$	M1
	$=\alpha\beta+\frac{1}{\alpha\beta}+\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	A1 (M1 on ePen)
	$=\frac{4}{3}+\frac{3}{4}+\frac{\frac{1}{9}-\frac{8}{3}}{\frac{4}{3}}=\frac{1}{6}$ oe	dM1 A1 (4) **[7]**

Part	Mark	Additional Guidance
(a)	B1	$\alpha+\beta=\frac{1}{3}$, $\alpha\beta=\frac{4}{3}$ can be explicit or implicitly used later
	M1	For an initial expression correctly adding the roots of g(x) and an attempt to simplify. **Minimum attempt must involve terms** $\alpha+\beta$ **and an attempt to simplify, bringing the fraction part to a common denominator** $\alpha\beta$ **written as a single fraction.**
	A1*	cso*
(b)	M1	For multiplying the roots of g(x). **Minimum attempt must involve a correct multiplication of the brackets with terms** $\alpha\beta+\frac{1}{\alpha\beta}$ **and an attempt to bring the fraction part to a common denominator** $\alpha\beta$
	A1 (M1 ePen)	Correct expression as shown.
	dM1	Substitutes their values for $\alpha+\beta$ and $\alpha\beta$ **correctly** into their expression (this expression must be ready for substitution of **both** $\alpha+\beta$ **and** $\alpha\beta$)
	A1	$\frac{1}{6}$ oe

Question number	Scheme	Marks
9	$\dfrac{2^{4x}}{2^{3y}} = \dfrac{1}{2^2}$	M1
	$2^{4x-2y} = 2^{-2}$ $(\rightarrow\ 4x - 3y = -2)$	dM1
	$2^{2x}2^{y} = 2^4$	M1
	$2^{2x+y} = 2^4$ $\rightarrow\ (2x + y = 4)$	dM1
	A fully correct method using for solving simultaneously leading to either $10x = 10$ **or** $5y = 10$ $4x - 3y = -2 \Rightarrow 10x = 10$ or $4x - 3y = -2 \Rightarrow 5y = 10$ $6x + 3y = 12$ $4x + 2y = 8$	ddddM1
	$y = 2$ $x = 1$	A1 A1 **[7]**
	Alternative Method	
	$4^x = \dfrac{16}{2^y}$	M1
	$\dfrac{4^{2x}}{8^y} = \dfrac{1}{4}$	M1
	$\left(\dfrac{16}{2^y}\right)^2 \times \dfrac{1}{8^y} = \dfrac{1}{4}$	ddM1
	$8^y \times 2^{2y} = 4 \times 16^2$	dddM1
	$2^{3y} \times 2^{2y} = 2^2 \times 2^8$	ddddM1
	$(2^{5y} = 2^{10})$ $y = 2$	A1
	$(4^x \times 4 = 16)$ $x = 1$	A1

Part	Mark	Additional Guidance
(a)	M1	For correctly changing any two indices into powers of 2 and simplifying. Accept any two of 2^2 or 2^{4x} or 2^{3y}
	dM1	**Dependent on previous method mark.** A fully correct method using index laws to simplify their expressions as powers of 2 and an attempt to write this as a linear equation.
	M1	For correctly changing both indices to powers of 2, as shown
	dM1	**Dependent on previous method mark.** A fully correct method using index laws to simplify their expressions as powers of 2 and an attempt to write this as a linear equation.
	ddddM1	**Dependent on all previous method marks**
	A1	$y = 2$
	A1	$x = 1$
ALT	M1	For a correct rearrangement of the 2nd equation as shown
	M1	For converting the 16^x into 4^{2x} as shown.
	ddM1	**Dependent on both previous method marks.** Substitution of $\frac{16}{2^y}$ into the second equation, this need not be fully simplified.
	dddM1	**Dependent on all previous method marks.** An attempt to rearrange the equation, must have at least one side of the equation shown correct.
	ddddM1	**Dependent on all previous method marks.** An attempt to convert **all** into powers of 2, must see at least 2 of $2^{3y}, 2^{2y}, 2^2, 2^8$ correctly written.
	A1	$y = 2$
	A1	$x = 1$

Question number	Scheme	Marks
10 (a)	$(x+\frac{\pi}{3})=\frac{\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{7\pi}{3}$	M1
	$(x+\frac{\pi}{3})=\frac{\pi}{3}$ and $\frac{2\pi}{3}$ and $\frac{7\pi}{3}$	A1
	$x=0,\frac{\pi}{3},2\pi$	A1 (3)
(b)	$\tan\theta=-\frac{5}{3}$	M1
	$\theta=-59^\circ, -239^\circ, 121^\circ, 301^\circ$	M1 A1 (3)
(c)	$1+\sin 2y-2(1-\sin^2 2y)=0$	M1
	$2\sin^2 2y+\sin 2y-1=0$	A1
	$(\sin 2y+1)(2\sin 2y-1)=0$	
	$\sin 2y=-1$ or $\sin 2y=\frac{1}{2}$	dM1
	$2y=-90^\circ, (30^\circ), (150^\circ), -330^\circ, -210^\circ$	A1
	$y=-45^\circ, -105^\circ, -165^\circ$	A1 (5)
		[11]

Part	Mark	Additional Guidance
(a)	M1	Any one of the three indicated angles, radians only, ignore any other angles.
	A1	For all three indicated angles, ignore other angles out of the range $\frac{\pi}{3}\le x+\frac{\pi}{3}\le\frac{7\pi}{3}$
	A1	For all three angles, ignore angles out of range, A0 if additional angles in range.
(b)	M1	For $\tan\theta=k.\ \ k\neq0,\ k\neq\pm1$
	M1	Any one correct value, does not need to be to the nearest degree. Allow one correct value to imply the first M1. Ignore any other angles.
	A1	For all four angles, ignore angles out of range, A0 if additional angles in range. All four angles must be given to the nearest degree.
(c)	M1	For the correct use of $1-\sin^2 2y$ in the equation on the left or right side, equation doesn't need to be = 0.
	A1	Correct 3TQ, must be = 0 and a valid attempt to solve leading to sin 2y =
	dM1	$\sin 2y=-1$ or $\sin 2y=\frac{1}{2}$ (allow sin2y = a and b from a valid attempt to solve their 3TQ). Allow sin2y to be x or any other variable.
	A1	For minimum of 3 of the 5 values shown, (including the ones in brackets), ignore other angles outside the range -360° ≤ 2y ≤ 360°. Allow sight of 270 if -90° present.
	A1	For all 3 values shown. Ignore extras out of range. **Rounding answers (where accuracy is specified in the question)** Penalise only once per question for failing to round as instructed - ie giving more digits in the answers.

Question number	Scheme	Marks
11 (a)	$\mathbf{b}-\mathbf{a}$	B1 (1)
(b)	$\overrightarrow{OZ} = \overrightarrow{OB} + \lambda\,\overrightarrow{BX} (= \mathbf{b} + \lambda(-\mathbf{b}+2\mathbf{a}))$	M1
	$= (1-\lambda)\mathbf{b} + 2\lambda\mathbf{a}$	A1
	$\overrightarrow{OZ} = \overrightarrow{OA} + \mu\,\overrightarrow{AY} (= \mathbf{a} + \mu(-\mathbf{a}+3\mathbf{b}))$	M1
	$= (1-\mu)\mathbf{a} + 3\mu\mathbf{b}$	A1
	$(1-\lambda)\mathbf{b} + 2\lambda\mathbf{a} = (1-\mu)\mathbf{a} + 3\mu\mathbf{b}$	ddM1
	$2\lambda = 1-\mu$ $3\mu = 1-\lambda$	A1
	$3(1-2\lambda) = 1-\lambda$ or $2(1-3\mu) = 1-\mu$	M1
	$\lambda = \frac{2}{5}$ or $\mu = \frac{1}{5}$	A1
	$\overrightarrow{OZ} = \frac{1}{5}(4\mathbf{a}+3\mathbf{b})$ See notes regarding alternatives	A1 (9)
(c)	$\overrightarrow{OM} = p\text{"}\frac{1}{5}(4\mathbf{a}+3\mathbf{b})\text{"}$ and $\overrightarrow{OM} = 2\mathbf{a} + q(-2\mathbf{a}+3\mathbf{b})$	M1
	$\frac{4p}{5} = 2-2q$ and $\frac{3p}{5} = 3q$	M1
	(Solving these equations leads to $p = \frac{5}{3}$)	
	$\overrightarrow{OM} = \frac{1}{3}(4\mathbf{a}+3\mathbf{b})$	A1 (3)
		[13]

Part	Mark	Additional Guidance
(a)	B1	For the indicated vector
(b)	M1	For any correctly written vector path, must include a parameter
	A1	For the vector shown
	M1	For any correctly written vector path, must include a parameter
	A1	For the vector shown
	ddM1	Equates their 2 vectors – this mark may be implicit in the candidate equating the two components of their 2 vectors, **dependent on the first two method marks.**
	A1	Correct equations as shown
	ddM1	Full and correct method to solve their two simultaneous equations, either by substitution as shown or by elimination. There must be no errors in the method to eliminate λ or μ, **dependent on the first two method marks.**
	A1	Correct value for λ or μ
	A1	Correct vector.
	There are a number of alternatives for part b, all marked in the same way. Examples: ALT1 $\mu \overrightarrow{XB}$ and $\lambda \overrightarrow{AY}$ (must use to get $\overrightarrow{AB}$ for 2nd A1) equate 2 vectors for $\overrightarrow{AB}$ ALT2 $\overrightarrow{AZ} = \overrightarrow{AX} + \mu \overrightarrow{XB}$ M1A1 $\overrightarrow{AZ} = \lambda \overrightarrow{AY}$ M1A1 equate 2 vectors for $\overrightarrow{AZ}$ ddM1 The A1 ddM1 A1 A1 following these marks should all be marked in the same way as the main mark scheme.	
(c)	M1	For the two correct vectors shown, allow use of their $\overrightarrow{OZ}$
	dM1	Correctly equating the components of their vectors for $\overrightarrow{OZ}$ and arriving at a value for p or q
	A1	For the correct vector, as shown.
	Can also be done using other vectors eg finding two alternatives for $\overrightarrow{OM}$ Mark in the same way as main scheme.	

Question number	Scheme	Marks
12 (a)	$(2\cos x = 0)$ $\quad (x=)\dfrac{\pi}{2}$ or 90 °	B1 (1)
(b)	$(2\cos x = 2\sin x)$ $\quad \tan x = 1$	M1
	$x = \dfrac{\pi}{4}$ or 45°	A1 (2)
(c)	$\int_{(0)}^{(\text{"}\frac{\pi}{4}\text{"})}(2\sin x)\mathrm{d}x + \int_{(\text{"}\frac{\pi}{4}\text{"})}^{(\text{"}\frac{\pi}{2}\text{"})}(2\cos x)\mathrm{d}x$	M1
	$[-2\cos x]_0^{\frac{\pi}{4}} + [2\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	A1
	$\left(-\sqrt{2}+2\right)+\left(2-\sqrt{2}\right)=4-2\sqrt{2}$	dM1
	$= 4-\sqrt{8}$ cao	A1cao cso (4) **[7]**

Part	Mark	Additional Guidance
(a)	B1	For $\dfrac{\pi}{2}$ or 90 degrees. Can also be shown as a coordinate – ignore any incorrect y coordinate.
(b)	M1	For $\tan x = 1$
	A1	For $\dfrac{\pi}{4}$ or 45 degrees. Can also be shown as a coordinate – ignore any incorrect y coordinate.
(c)	M1	For both integrals correctly shown, with an addition sign between them. Limits need not be shown. Must be 2 integrals shown. Can't be shown as one integral with incorrect limits.
	A1	For both functions correctly integrated. Limits need not be shown.
	dM1	For their limits clearly and correctly substituted in or for the numerical expression(s) shown in the MS. If mark awarded for substitution, both integrated expressions must have both limits correctly substituted. 0 must be the lower limit on the first integral. Allow ft of their $\dfrac{\pi}{4}$ (must be the upper limit on the first integral and the lower limit on the second and can be in degrees) and their $\dfrac{\pi}{2}$ (must be the upper limit on the second integral and can be in degrees).
	A1	cao cso A0 if degrees used in part c
		Note, can also be completed as either integral doubled – symmetry. M1 – correct integral stated with multiply by 2 evident or implicit later. A1 correctly integrated dM1 – as for main scheme, the multiply by 2 must be clearly shown. A1 as main scheme