Mark Scheme (Results)

January 2023

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  You must always check the working in the body of the script (and on any diagrams) irrespective of whether the final answer is correct or incorrect and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$, where $|pq|=|c|$ leading to $x=\ldots.$

$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\ldots.$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\ldots.$

4. Use of calculators

Unless the question specifically states 'show' or 'prove' accept correct answers from no working. If an incorrect solution is given without any working do not award the Method mark.

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

International GCSE Further Pure Mathematics – Paper 1 Mark scheme

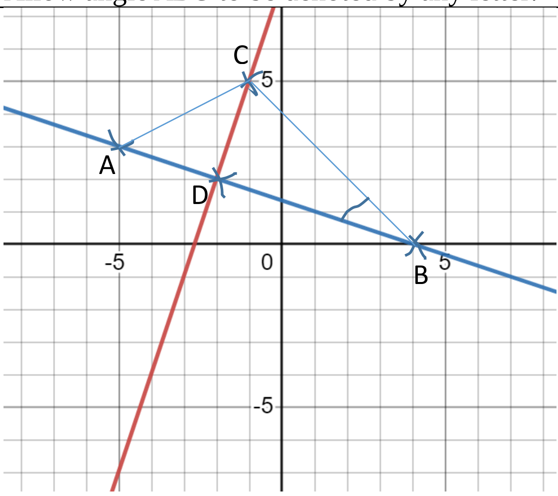
Paper 1		
Question number	Scheme	Marks
1	$a+9d+a+10d+a+11d$ or $3a+30d=129$ oe $a+18d+a+19d+a+20d=237$ or $3a+57d=237$ oe	M1 M1
	$27d$ = 108 oe (d = 4)	M1
	$(a_1=)3$	A1 (4)
ALT	$a_{10}+a_{10}+d+a_{10}+2d$ or $3a_{10}+3d=129$ oe $a_{10}+9d+a_{10}+10d+a_{10}+11d=237$ or $3a_{10}+30d=237$ oe	M1 M1
	$27d$ = 108 oe (d = 4)	M1
	$(a_1=)3$	A1
		Total 4 marks

Mark	Additional Guidance
M1	For fully correct use of $a+(n-1)d$ in an equation for the 10^{th}, 11^{th} and 12^{th} terms **OR** the 19^{th}, 20^{th} and 21^{st} terms simplified or unsimplified oe
M1	For fully correct use of $a+(n-1)d$ in an equation for the 10^{th}, 11^{th} and 12^{th} terms **AND** the 19^{th}, 20^{th} and 21^{st} terms simplified or unsimplified oe
M1	For an attempt to solve their equations simultaneously, allow one processing error. Must eliminate one variable and arrive at a value for a or d.
A1	For $(a\ /\ a_1=)3$
	Allow mixed use of a or a_1 throughout

Question number	Scheme	Marks
2 a	(Gradient of AB =) $\dfrac{0-3}{4+5}\left(=-\dfrac{1}{3}\right)$	M1
	Gradient of the perpendicular = $-\dfrac{1}{"-\frac{1}{3}"}(=3)$	M1
	$y-5=-\dfrac{1}{"-\frac{1}{3}"}(x+1)$ or $y-5="3"(x+1)$ or $y=3x+8$	M1
	$3x-y+8=0$ or $-3x+y-8=0$ oe	A1 (4)
b	(Equation of AB =) $y-0="-\dfrac{1}{3}"(x-4)$ or $y=-\dfrac{1}{3}x+\dfrac{4}{3}$ oe	M1
	Solve $y-3x-8=0$ and $y-0=-\dfrac{1}{3}(x-4)$ oe simultaneously $0=\dfrac{10}{3}x+\dfrac{20}{3}$ oe	M1
	$x=-2$ $y=2$ or (-2, 2) *	A1 cso (3)
	Alternative for final 2 marks (substitutes (- 2, 2) into each equation)) Line l is $3x-y+8=0$ When $x=-2$ and $y=2$, $-6-2+8=0\Rightarrow D$ lies on l AB is $y=-\dfrac{1}{3}\ x-4$ When $x=-2$ and $y=2$, $-\dfrac{1}{3}\ -2-4\ =2=y\Rightarrow D$ lies on AB	{M1}
	So l and AB intersect at $-2, 2$ *	{A1 cso} (3)
c	Midpoint of AB = $\left(\dfrac{-5+4}{2},\dfrac{3(+0)}{2}\right)=\left(-\dfrac{1}{2},\dfrac{3}{2}\right)$	B1 (M1 on ePen)
	$\neq(-2,2)$ So not perpendicular bisector of AB *	B1 (A1 on ePen) cso (2)
ALT	$BD=\sqrt{(4--2)^2+(0-2)^2}\left(=\sqrt{40}\right)$ or $AD=\sqrt{(-5--2)^2+(3-2)^2}\left(=\sqrt{10}\right)$	M1
	$BD=\sqrt{40}(=2\sqrt{10})$ and $AD=\sqrt{10}$ $BD\neq AD$ so not the perpendicular bisector	A1cso (2)

d	$BD = \sqrt{(4 - -2)^2 + (0-2)^2}\left(=\sqrt{40}\right)$ oe	M1
	$CD = \sqrt{(-1 - -2)^2 + (5-2)^2}\left(=\sqrt{10}\right)$ oe	M1
	$\left(\tan\angle ABC =\right)\dfrac{"\sqrt{(-1 - -2)^2 + (5-2)^2}"}{"\sqrt{(4 - -6)^2 + (0-2)^2}"}$ or $\dfrac{\sqrt{10}}{\sqrt{40}}$	ddM1
	$= \dfrac{1}{2}$	A1 (4)
ALT 1	Uses cosine rule in triangle *ABC* $AB = \sqrt{(-5-4)^2 + (3-0)^2}(=\sqrt{90} = 3\sqrt{10})$ or $BC = \sqrt{(4 - -1)^2 + (0-5)^2}(=\sqrt{50} = 5\sqrt{2})$ or $AC = \sqrt{(-1 - -5)^2 + (5-3)^2}(=\sqrt{20} = 2\sqrt{5})$ may also find expressions for $(AB)^2, (BC)^2$ or $(AC)^2$	M1
	$AB = \sqrt{(-5-4)^2 + (3-0)^2}(=\sqrt{90} = 3\sqrt{10})$ and $BC = \sqrt{(4 - -1)^2 + (0-5)^2}(=\sqrt{50} = 5\sqrt{2})$ and $AC = \sqrt{(-1 - -5)^2 + (5-3)^2}(=\sqrt{20} = 2\sqrt{5})$ may also find expressions for $(AB)^2, (BC)^2$ and $(AC)^2$	M1
	$\left(\cos ABC =\right)\dfrac{("\sqrt{50}")^2 + ("\sqrt{90}")^2 - ("\sqrt{20}")^2}{2\times"\sqrt{50}"\times"\sqrt{90}"}\left(=\dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}\right)$	ddM1
	$\left(\tan ABC\right) = \dfrac{1}{2}$	A1
		Total 13 marks

Part	Mark	Additional Guidance
(a)	M1	For correctly finding the gradient of *AB*, need not be simplified.
	M1	For finding the negative reciprocal of their gradient of *AB*, need not be simplified.
	M1	For a fully correct method using their negative reciprocal to find the equation of the straight line. Allow use of their $-\frac{1}{"-\frac{1}{3}"}$ which can be unsimplified or processed incorrectly, but must follow from a clear attempt to have found the negative reciprocal of their gradient for *AB*. If $y = mx + c$ is used, there must be a fully correct substitution and a fully correct rearrangement to find c
	A1	For $3x - y + 8 = 0$ or $-3x + y - 8 = 0$ oe so long as integer coefficients and = 0.
(b)	M1	For writing the equation of *AB*, the equation need not be simplified, but must be fully correct.
	M1	For an attempt to solve their equations simultaneously, allow one processing error. Must eliminate one variable and arrive at a value for *x* or *y*.
	A1 cso*	For obtaining the required result, no errors.
	ALT	**Final 2 marks**
	M1	For writing the equation of *AB*, the equation need not be simplified, but must be fully correct.
	M1	For a fully correct substitution of (–2, 2) into a correct equation for *AB* to show that *D* lies on *AB* **OR** a fully correct substitution of (–2, 2) into **their** equation for line *l*
	A1 cso*	For a fully correct substitution of (–2, 2) into a correct equation for *AB* to show that *D* lies on *l* **AND** a fully correct substitution of (–2, 2) into a correct equation for line *l* to show that *D* lies on *l*. An appropriate (minimal) conclusion must also be written, this can be as simple as # or 'shown'. No errors.
(c)	B1 (M1 on ePen)	For either coordinate of the midpoint correct
	B1 (A1 on ePen) cso*	For both coordinates correct and an appropriate (minimal) conclusion, which must be correct, can be as simple as # or 'shown'.
ALT	M1	For a correct method to calculate *AD* or *BD*
	A1 cso*	For correct values for *AD* and *BD*, accept decimals and a (minimal) conclusion, can be as simple as # or 'shown'.
(d)	M1	For the correct method to calculate *BD* (May be seen in (c))
	M1	For the correct method to calculate C*D*
	ddM1	For $(\tan\angle ABC =)\frac{\sqrt{(-1--2)^2+(5-2)^2}}{\sqrt{(4--6)^2+(0-2)^2}}$ or $\frac{\sqrt{10}}{\sqrt{40}}$ allow their *BD* and *CD*. Dependent on the previous 2 method marks.
	A1	For $(\tan\angle ABC) = \frac{1}{2}$
If you see a method using the gradient of *BC and* tan (*A* – *B*) – PLEASE SEND TO REVIEW		

ALT 1	**M1**	Correct method to calculate one of the required lengths
	M1	Correct method to calculate all of the required lengths
	ddM1	Correct substitution into the cosine rule to give the expression shown, allow use of their lengths. Must obtain a correct expression for cos*ABC* = , using their values. Dependent on the previous 2 method marks.
	A1	For $\left(\tan \angle ABC\right) = \frac{1}{2}$
Because of the wording of the question, in not specifically asking for an exact value, we are going to allow a range for the value of $\tan \angle ABC$ as follows: $0.49989 \le \tan \angle ABC \le \frac{1}{2}$		
If you see the candidate using the ratio for a right angled triangle - but they are using the wrong pairing of lengths, (perhaps *AB*, *BC* or *AC*) then this is to be marked using the main scheme, and is unlikely to be awarded any marks unless they have correctly calculated *BD* or *CD*. To be clear - If the candidate states correct values for *AB*, *BC* or *AC* but goes onto to use a tan ratio - this cannot be awarded any marks using the ALT mark scheme, because they are not using the correct cosine rule and with the tan ratio they should be using at least one of the correct sides, *BD* or *CD*. If you see a clear attempt to use the cosine rule, please use this ALT marking scheme. A special case is when the candidate has not made it clear which route they were intending to take - tan ratio (right angled triangle ratio) or cosine rule. If they have all lengths stated correctly, they could be awarded M1 M1 using either scheme.		
Further ALTS		If you see a valid method, leading to a fully correct exact value for tan*ABC*, please award 4 marks. **If the value for tan*ABC* is incorrect or approximate and there is some partially correct working, PLEASE SEND TO REVIEW**. Allow angle *ABC* to be denoted by any letter.
		 For reference $BD = \sqrt{40}(= 2\sqrt{10})$ $AD = \sqrt{10}$ $AB = \sqrt{90} = 3\sqrt{10}$ $BC = \sqrt{50} = 5\sqrt{2}$ $\sin ABC = \frac{\sqrt{5}}{5}$ $\cos ABC = \frac{2\sqrt{5}}{5}$

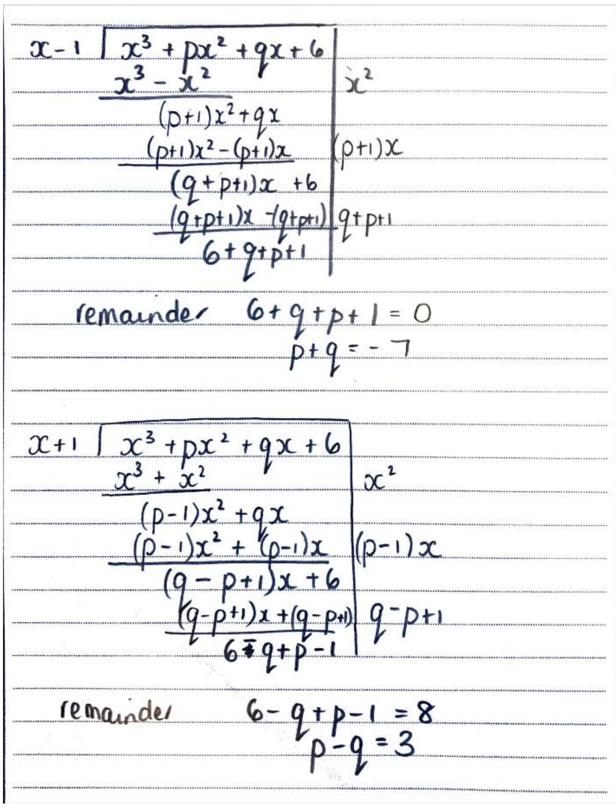
Question number	Scheme	Marks
3 a	$\frac{a}{-2}=4 \Rightarrow a=-8$	M1 A1 (2)
b	$x=\frac{1}{2}$	B1 (1)
c (i)	$\left(\frac{3}{-"-8"},0\right)\left(=\left(\frac{3}{8},0\right)\right)$	B1 ft
c (ii)	$(0,3)$	B1 (2)
d	Asymptote drawn at $y=4$. The asymptote must either be clearly labelled with its equation or pass through the *y* axis with 4 clearly labelled.	B1
	Asymptote with equation drawn at $x=\frac{1}{2}$ The asymptote must either be clearly labelled with its equation or pass through the *x* axis with $\frac{1}{2}$ labelled.	B1 ft
	Correct curve drawn with two branches	B1 ft
	"$\frac{3}{8}$" and "3" labelled on *x* and *y* axes respectively	B1 ft (4)
		Total 9 marks

Part	Mark	Additional Guidance
(a)	**M1**	For $\frac{a}{-2}=4$
	A1	For $a=-8$
	Note: it is acceptable if a candidate uses a suitably large value for *x* and deduces the correct value for *a*. M1 A1	
(b)	**B1**	For $x=\frac{1}{2}$
(c)(i)	**B1 ft**	For $\left(\frac{3}{-"-8"},0\right)\left(=\left(\frac{3}{8},0\right)\right)$ ft their value from (a). Allow omission of brackets. This mark is awarded for the point at which you see $\left(\frac{3}{-\text{their } a},0\right)$ May be written *x* =
(ii)	**B1**	For $(0,3)$. Allow omission of brackets. May be written *y* =
If there is no labelling of (i) and (ii), it must be unambiguously indicated which coordinate crosses the *y* axis and which crosses the *x* axis. If any doubt, do not award the marks.		
(d)	**B1**	For asymptote with equation drawn at $y=4$. The line must be labelled with the equation or must clearly pass through the *y*-axis labelled as 4. There must be at least one branch of the curve present, which must not cross or bend back from the asymptote.
	B1 ft	For asymptote with equation drawn at $x=\frac{1}{2}$, ft their answer from part b The line must be labelled with the equation or must clearly pass through the *x*-axis labelled as 0.5. There must be at least one branch of the curve drawn, which must not cross or bend back from the asymptote.

	B1 ft	For correct curve drawn with two branches, in the correct quadrants, ft their answers from part a and b. The curve must be drawn in quadrants 1 and 3.
	B1 ft	For a curve passing through "$\frac{3}{8}$" and "3" labelled on *x* and *y* axes respectively, ft their answers from part a, b and c.

Note: answers for parts a, b and c cannot be awarded marks for correct values labelled on the sketch for part d. For all parts where coordinates are requested, allow *x* = and *y* =

Question number	Scheme	Marks
4 a Mark parts (i) and (ii) together	$(\mathrm{f}(1)=)1^3+p\times1^2+q\times1+6=0$ oe or $(\mathrm{f}(-1)=)(-1)^3+p\times(-1)^2+q\times(-1)+6=8$ oe	M1
	$p+q=-7$ oe	A1
	$p-q=3$ oe	A1
	Solving simultaneously $2p=-4$ or $2q=-10$	M1
	$p=-2$ *	A1 cso
	and $q=-5$	B1 (A1 on ePen) cso
		(6)
b	$(\mathrm{f}(x)=x^3-2x^2-5x+6=0)$	
	$(x-1)(x^2-x-6)=0$	M1
	$[(x-1)](x+2)(x-3)=0$	M1
	$x=1,-2,3$	A1
		(3)
		Total 9 marks



<table>
<tr><th>Part</th><th>Mark</th><th>Additional Guidance</th></tr>
<tr><td rowspan="7">(a)</td><td>M1</td><td>For a fully correct substitution of either 1 into f(x) = 0 OR –1 into f(x) = 8 as shown. The bracketing for the substitution of –1 must be correct, but may be recovered later. A correct equation can imply this mark.</td></tr>
<tr><td>A1</td><td>For a fully correct substitution of 1 into f(x) = 0 AND –1 into f(x) = 8 as shown. The bracketing for the substitution of –1 must be correct, but may be recovered later. A correct equation can imply this mark.</td></tr>
<tr><td>A1</td><td>For $p+q=-7$ and $p-q=3$ oe
The equations do not need to be simplified.</td></tr>
<tr><td colspan="2">Note, if students choose to do long division, to be comparable with the main scheme, this must be fully correct and complete, to allow them to arrive at the correct equations to be awarded the method mark. See the example included. The accuracy marks can be then awarded as stated above. There is an example under the main mark scheme.
If in any doubt, please send to review.</td></tr>
<tr><td>M1</td><td>For an attempt to solve their equations simultaneously, allow one processing error. Must eliminate one variable and arrive at a value for p or q.</td></tr>
<tr><td>A1 cso</td><td>For $p=-2$</td></tr>
<tr><td>B1 (A1 on ePen)</td><td>$q=-5$
Note this is an independent accuracy mark, allowing students to use the given value of p to find q.</td></tr>
<tr><td rowspan="3">(b)</td><td>M1</td><td>For $(x-1)(x^2 \pm Ax-6)=0 \quad A\neq 0$
This mark can be awarded for sight of $x^2 \pm Ax-6 \quad A\neq 0$</td></tr>
<tr><td>M1</td><td>For $[(x-1)](x+2)(x-3)=0$
For this mark, for the correct or ft their quadratic:
Allow any minimally acceptable attempt to solve the quadratic, by factorising, completing the square or use of the quadratic formula – see general guidance, leading to two values of x in addition to x = 1. It is not necessary to see = 0.</td></tr>
<tr><td>A1</td><td>For $x=1,-2,3$
If 3 three correct solutions appear with no working M1 M1 A1 as the question has not stated show working or solve algebraically.</td></tr>
</table>

Question number	Scheme	Marks
5 a	$\left(y=1+\frac{k}{2}x\Rightarrow\right)$ eg $kx^2-x\left(1+\frac{k}{2}x\right)+(k+1)x-1=0$ or eg $\frac{kx^2+(k+1)x-1}{x}=\frac{k}{2}x+1$	M1
	$kx^2-x-\frac{k}{2}x^2+kx+x-1=0$ or $2kx^2+2(k+1)x-2=kx^2+2x$ oe And an attempt to simplify must be made.	M1
	$kx^2+2kx-2=0$ oe	A1
	$\left(b^2-4ac=0\Rightarrow\right)("2k")^2-4("k")("-2")=0$ or $(x+1)^2=1+\frac{2}{k}$	M1
	$4k(k+2)=0$ or $k+2=0$or $1+\frac{2}{k}=0$	dM1
	$k=-2$	A1 (6)
ALT **Final 3 marks**	Realises if the line intersects the curve only once, it is a tangent at point A – final 3 marks	
	$y=\frac{kx^2+(k+1)x-1}{x}=\left(kx+k+1-\frac{1}{x}\right)$ and $\left(\frac{dy}{dx}=\right)\frac{k}{2}$	M1
	$\left(\frac{dy}{dx}=\right)k(+0)+\frac{1}{x^2}=\frac{k}{2}\Rightarrow x^2=-\frac{2}{k}$	
	$k\left(-\frac{2}{k}\right)+2k\sqrt{-\frac{2}{k}}-2=0\Rightarrow 2k\sqrt{-\frac{2}{k}}=4\Rightarrow 4k^2\left(-\frac{2}{k}\right)=16\Rightarrow -8k=16$	dM1
	k = - 2	A1
b	When $k=-2$ $-2x^2-4x-2=0$ or $(x+1)^2=1+\frac{2}{-2}$	
	$(x+1)^2=0$	M1
	$x=-1\Rightarrow y=2$ So $(-1,2)$	A1 (2)
		Total 8 marks

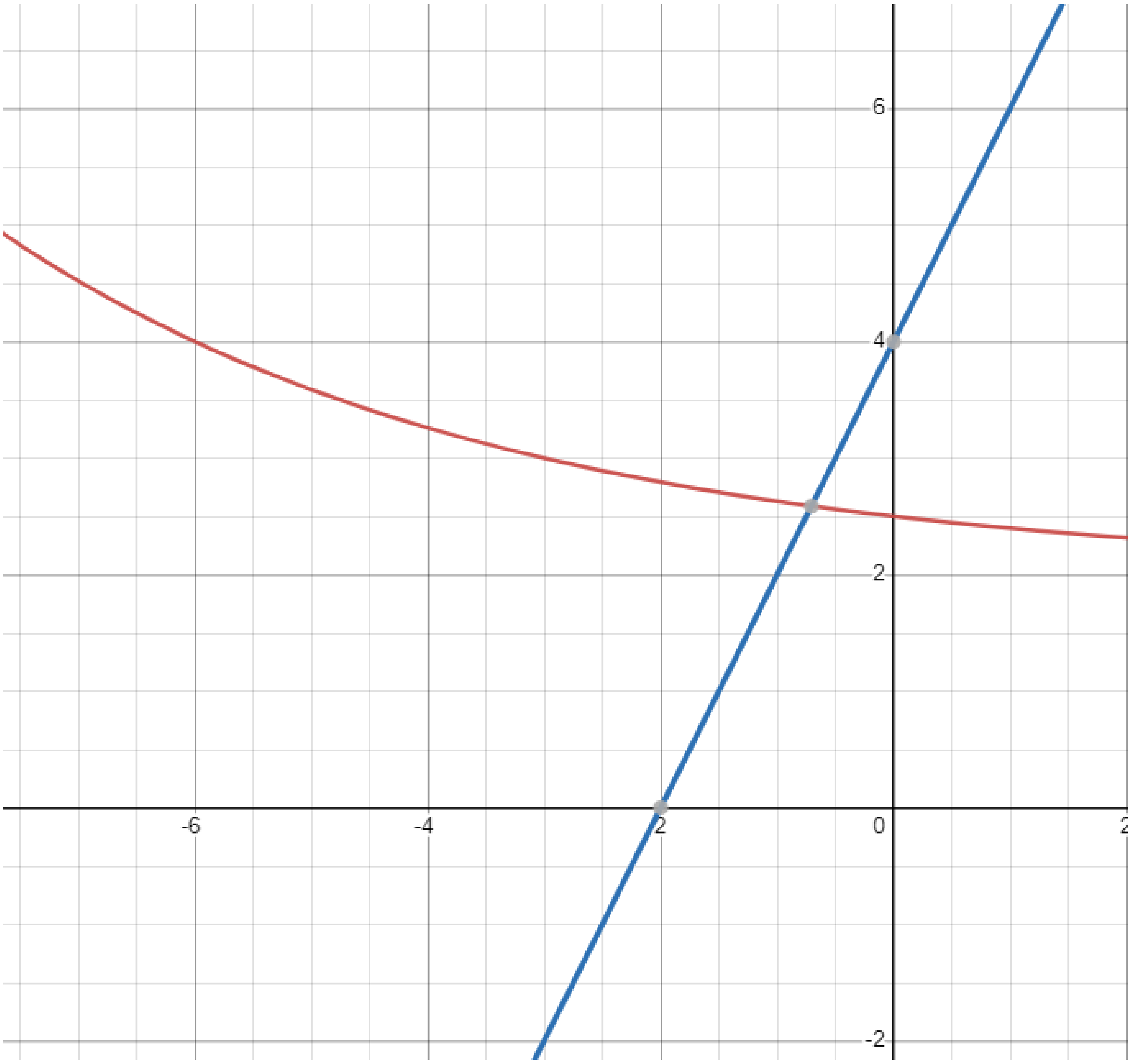
Part	Mark	Additional Guidance
(a)	M1	For correctly substituting $y=1+\frac{k}{2}x$ into $kx^2-xy+(k+1)x=1$
	M1	For a complete expansion of $x\left(1+\frac{k}{2}x\right)$ and $(k+1)x$. Allow one error in the expansion. For multiplying the equation throughout by 2x or -2x, x or -x to eliminate any algebraic denominators, allow one processing error. There must be an attempt to simplify follow either of these steps. If any other manipulation is seen, full marks can be awarded if the steps lead to a correct equation or a three term quadratic with one processing error made. If there is relevant work, not leading to either of these, please send to review.
	A1	For a correct three term quadratic. oe
	M1	For **clear and correct** use of $b^2-4ac=0$ or a **full and correct** completing the square process, using their expressions for a, b and c. $a,b,c\neq 0$
	dM1	For factorising or any attempt to solve their quadratic in k, or setting their $1+\frac{2}{k}=0$ See general guidance for acceptable attempt to solve a quadratic. For the case of a 2TQ with terms in k and k^2 (which a fully correct solution delivers), we need to see a correct factorisation in the case of this mark or, as the candidate is told k is no zero, they may also correctly divide throughout by k. Dependent on the previous method mark. Allow this mark to be implied if k is correct. Otherwise, the method must be shown to gain this mark.
	A1	$k=-2$, must dismiss k = 0, if found.
ALT	**States and use the gradient of C is $\frac{k}{2}$.**	
	M1	As main scheme.
	M1	As main scheme.
	A1	As main scheme.
	M1	Differentiates C and puts the gradient = $\frac{k}{2}$ to get an equation of the form $mk+\frac{n}{x^2}=\frac{k}{2}$ $m,n\neq 0$ (doesn't need to be simplified) but must lead to an expression of the form $x^2=-\frac{l}{k}$ $l\neq 0$
	dM1	Substitutes their expression for x^2 into their quadratic equation and rearranges to give an expression of the form $-pk=q$ or $pk=-q$ $p>0,k>0$
	A1	k = -2 (must be the only solution given).
(b)	M1	For correct substitution of their value of k into their 3 term quadratic from part a and a minimally acceptable attempt to solve for x. See general guidance.
	A1	For $(-1,2)$ or x = – 1 and y = 2

Question number	Scheme	Marks
6 a	$p=2 \quad q=\dfrac{3}{8}$	B1 B1 (2)
b	$(2)\left[1+\frac{1}{3}\left(\text{"}\frac{3}{8}\text{"}x\right)+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2!}\left(\text{"}\frac{3}{8}\text{"}x\right)^2\right]=2+\frac{1}{4}x-\frac{1}{32}x^2$	M1 A1ft A1 (3)
c	$\left(\sqrt[3]{9}=\sqrt[3]{8+1}\Rightarrow x=\frac{1}{3}\right)$ So $\text{"}2\text{"}+\text{"}\frac{1}{4}\text{"}\times\text{"}\frac{1}{3}\text{"}-\text{"}\frac{1}{32}\text{"}\times\left(\text{"}\frac{1}{3}\text{"}\right)^2\left(=\frac{576+24-1}{288}\right)=\frac{599}{288}$ *	M1 A1 cso (2)
		Total 7 marks

Part	Mark	Additional Guidance
(a)	**B1**	For $p=2$ **or** $q=\dfrac{3}{8}$ condone $8^{\frac{1}{3}}$
	B1	For $p=2$ **and** $q=\dfrac{3}{8}$ condone $8^{\frac{1}{3}}$
(b)	**M1**	For an attempt to expand $(1+qx)^{\frac{1}{3}}$ with their value of q up to the term in x^2 It is not necessary to see p at this stage. The definition of an attempt is as follows: • The first term must be 1 • The next term must be correct for their value of q • The powers of qx must be correct eg $(qx)^2$ • The denominators must be correct Simplification not required. Do not allow missing brackets unless recovered later – this is a general point of marking. Ignore any terms with powers higher than 2.
	A1ft	For both algebraic terms fully correct and unsimplified in the expansion of $(1+qx)^{\frac{1}{3}}$ for their value of q. It is not necessary to see p at this point. Ignore any terms with powers higher than 2.
	A1	For all 3 terms correct, all simplified. Ignore any terms with powers higher than 2
If there are any other methods used – send this to review please.		
(c)	**M1**	For correct substitution of $x=\dfrac{1}{3}$ into their expansion, which must have at least 2 terms, including a term in x^2
	A1cso*	Obtains the given result

Question number	Scheme	Marks
7 a	x: -6, -5, -4, –3, –2, –1, 0 y: 4, 3.59, 3.26, **3**, **2.79**, **2.63**, 2.5	B2 (2)
b	Points plotted within a half square	B1 ft
	Joined with a smooth curve	B1 ft (2)
c	$\left(\log_2 (2x+2)^3 + x + 3 = 0\right)$	
	$3\log_2(2x+2) = -x-3$ or $\log_2(2x+2) = \frac{-x}{3} - 1$	M1
	$\frac{\log_{0.5}(2x+2)}{\log_{0.5}2} = -\frac{x}{3} - 1$ or $2x+2 = 2^{-\frac{x}{3}-1}$ or $\frac{1}{2x+2} = 2^{\frac{x}{3}+1}$	M1
	$\left(\log_{0.5}(2x+2) = \frac{x}{3} + 1 \Rightarrow\right) 2x+2 = 0.5^{\frac{x}{3}+1}$	M1
	$2x+4 = 0.5^{\left(\frac{x}{3}+1\right)} + 2$	A1
	$y = 2x+4$ drawn	M1
	Intersect at $x = –0.8 / –0.7$	A1 (6)
ALT	$3\log_2(2x+2) = -x-3$ or $\log_2(2x+2) = \frac{-x}{3} - 1$	M1
	$\frac{\log_{0.5}(2x+2)}{\log_{0.5}2} = -\frac{x}{3} - 1\left(\Rightarrow \log_{0.5}(2x+2) = \frac{x}{3} + 1\right)$	M1
	$\left(y - 2 = 0.5^{\left(\frac{x}{3}+1\right)} \Rightarrow\right)\log_{0.5}(y-2) = \frac{x}{3} + 1 \Rightarrow \log_{0.5}(y-2) = \log_{0.5}(2x+2)$	M1
	$y - 2 = 2x + 2$	A1
	As main scheme	M1 A1
		Total 10 marks

Part	Mark	Additional Guidance
(a)	B2	For all 3 values in the table correct to 2 d.p. Allow 3.0 or 3.00 for "3" (B1 for 2 values)
(b)	B1 ft	For all of the points plotted within half a square, allow use of their values. Points must be checked carefully, including using the zoom tool on ePen if necessary.
	B1 ft	For all of their points joined with a smooth curve. The curve must pass through each of their points to within half a square. Be cautious to not award this mark if straight lines are drawn between the points plotted.
(c)	M1	For use of $\log_a x^k = k\log_a x$ and an attempt to rearrange. Accept an equation of the form $\pm 3\log_2(2x+2) = \pm x \pm 3$ or $\pm\log_2(2x+2) = \frac{\pm x}{3} \pm 1$
	M1	For correctly dividing by 3 (if not already done) and correct use of $\log_b a = \frac{\log_{0.5}(a)}{\log_{0.5}(b)}$ with their equation to get an expression of the form $\frac{\log_{0.5}(2x+2)}{\log_{0.5} 2} = \pm\frac{x}{3} \pm 1$ or For correctly dividing by 3 (if not done) and correctly converting their equation to exponential form get an equation of the form $2x+2 = 2^{\pm\frac{x}{3}\pm 1}$ or $\frac{1}{2x+2} = 2^{\pm\frac{x}{3}\pm 1}$
	M1	For reaching an equation of the form $2x+2 = 0.5^{\pm\frac{x}{3}\pm 1}$
	A1	For $2x+4 = 0.5^{\left(\frac{x}{3}+1\right)} + 2$
	M1	For $y = 2x+4$ drawn. Correct line drawn can imply any of the 4 previous marks. Allow any line of $y = 2x + c$ to gain this mark. **If the candidate draws any line where *c* is other than 4, they must have gained the previous 3 method marks.**
	A1	For $x = -0.8 / -0.7$
ALT	M1	For use of $\log_a x^k = k\log_a x$ and an attempt to rearrange. Accept an equation of the form $\pm 3\log_2(2x+2) = \pm x \pm 3$ or $\pm\log_2(2x+2) = \frac{\pm x}{3} \pm 1$
	M1	For correctly dividing by 3 (if not already done) and correct use of $\log_b a = \frac{\log_{0.5}(a)}{\log_{0.5}(b)}$ with their equation to get an expression of the form $\frac{\log_{0.5}(2x+2)}{\log_{0.5} 2} = \pm\frac{x}{3} \pm 1$
	M1	For an attempt to rearrange and converting to log form to reach $\log_{0.5}(y \pm 2) = \pm\frac{x}{3} \pm 1 \Rightarrow \log_{0.5}(y \pm 2) = \pm\log_{0.5}(2x+2)$
	A1	For the correct equation show.
	M1 A1	As main scheme.



6
4
2
-2
-6
-4
2
0

Question number	Scheme	Marks
8 a	(Area) = $2xy+\frac{1}{2}x^2\left(\frac{1}{2}\right)\left(=2xy+\frac{1}{4}x^2=50\right)$	M1
	So $y=\frac{50-\frac{1}{4}x^2}{2x}\left(=\frac{25}{x}-\frac{1}{8}x\right)$	dM1
	$P=2x+4y+\frac{1}{2}x=\frac{5}{2}x+4y$	M1
	$=\frac{5}{2}x+4\left(\frac{25}{x}-\frac{1}{8}x\right)\Rightarrow P=2x+\frac{100}{x}$ *	M1 A1 cso (5)
b	$\left(\frac{dP}{dx}=\right)2-\frac{100}{x^2}$	M1 A1
	$\left(\frac{dP}{dx}=0\Rightarrow\right)2-\frac{100}{x^2}=0\Rightarrow x^2=50\Rightarrow x=5\sqrt{2}$ oe	M1 A1
	$\frac{d^2P}{dx^2}=\frac{200}{x^3}$ When $x=5\sqrt{2}$ $\frac{d^2P}{dx^2}=\frac{2}{5}\sqrt{2}(=0.565.....)>0$ therefore minimum	M1 A1 (6)
c	$P=2\left(5\sqrt{2}\right)+\frac{100}{5\sqrt{2}}=20\sqrt{2}$	M1 A1 (2)
		Total 13 marks

Part	Mark	Additional Guidance
(a)	**M1**	For writing an expression (may not be simplified) for the area in terms of x and y. Allow any expression of the form $xy + xy + ax^2$ oe where $a > 0$ This mark can be implied by an equation for A with the required $xy + xy + ax^2$ oe where $a > 0$
	dM1	For placing their expression for area = 50 and correctly rearranging to make y the subject. The expression for y does not need to be simplified.
	M1	For correctly writing an equation for perimeter in terms of x and y (may not be simplified). There must be a P =, though this may be seen later in their work. As long as P = appears by the end of the work, this mark can be awarded. Perimeter = instead of P = is acceptable for this mark
	M1	For correct substitution of their expression for y into their expression for ***P* or the Perimeter to reach an expression for *P* which must be in terms of *x* only. Simplification not required.**
	A1*cso	Obtains the given result with no errors. Perimeter = instead of P = is **NOT** acceptable for this mark
For this question, if we are confident any second solution provided is a re-start and the work the candidate wishes us to consider, this is the work can be marked.		
(b)	**M1**	For an attempt to differentiate the given expression for P wrt x It is not necessary to see $\frac{dP}{dx} =$ Award this mark for any expression of the form $2 \pm \frac{b}{x^2}$, $b \neq 0$ oe
	A1	For $2 - \frac{100}{x^2}$ oe
	M1	For placing their expression for $\frac{dP}{dx} = 0$ which must have a term in x^2, a correct rearrangement to solve the equation formed and an attempt to find a value for x
	A1	For $x = 5\sqrt{2}$ oe ignore $x = -5\sqrt{2}$
	M1	For $\frac{d^2P}{dx^2} = \pm\frac{c}{x^3}$ $c \neq 0$
	A1	For substituting in a correct value for x, evaluating correctly, stating > 0 and drawing a conclusion. This can be as simple as # or shown. Students may also argue that the second derivative has to be positive as x is positive. This must be a convincing argument that since x is positive the derivative is positive and then a valid minimal conclusion. Do not award this mark if a negative value of x is used and it is not discounted at some point.
	ALT M1	Final 2 marks For substitution of valid values for x either side of their value of x found for the minimum value into $\frac{dP}{dx} = 2 \pm \frac{b}{x^2}$, $b \neq 0$
	A1	For fully correct substitution of their values for x into a correct expression for a first derivative, correctly evaluating or arguing the sign of the derivative and a conclusion drawn. Do not award this mark if a negative value of x is used and it is not discounted at some point.
(c)	**M1**	For substitution of their value for $x = 5\sqrt{2}$ oe into the given expression for P
	A1	For $20\sqrt{2}$

Question number	Scheme	Marks
9	$\left(e^{2y}-x+2=0\Rightarrow\right)e^{2y}=x-2\Rightarrow 2y=\ln(x-2)$ or $y=\frac{1}{2}\ln(x-2)$	M1
	So $\ln(x+3)-\ln(x-2)=1$ oe	M1
	$\ln\left(\frac{x+3}{x-2}\right)=1$	M1
	$\frac{x+3}{x-2}=e$	M1
	$x+3=xe-2e\Rightarrow 3+2e=xe-x$ $3+2e=x(e-1)\Rightarrow x=\frac{3+2e}{e-1}=4.91$	ddddM1 A1
	$y=\frac{1}{2}\ln\left(\frac{3+2e}{e-1}-2\right)=0.53$	M1 A1 (8)
ALT 1	$e^{2y}+2=x\Rightarrow\ln(e^{2y}+2+3)-2y-1=0$	M1
	$\ln(e^{2y}+5)=2y+1\Rightarrow e^{2y+1}=e^{2y}+5$	M1
	$e.e^{2y}=e^{2y}+5$	M1
	$e^{2y}(e-1)=5$	M1
	$2y=\ln\left(\frac{5}{e-1}\right)\Rightarrow y=\frac{1}{2}\ln\left(\frac{5}{e-1}\right)$=0.53	ddddM1 A1
	$x=e^{\ln\left(\frac{5}{e-1}\right)}+2$ or $x=\left(\frac{5}{e-1}\right)+2$ = 4.91	M1 A1 (8)
ALT 2	$\ln(x+3)=2y+1\Rightarrow x=e^{2y+1}-3$	M1
	$e^{2y}-\left(e^{2y+1}-3\right)+2=0$	M1
	$e^{2y}-e^{2y+1}+5=0\Rightarrow e^{2y}-e.e^{2y}+5=0$	M1
	$(1-e)e^{2y}=-5$	M1
	$2y=\ln\left(\frac{-5}{1-e}\right)\Rightarrow y=\frac{1}{2}\ln\left(\frac{-5}{1-e}\right)$=0.53	ddddM1 A1
	Final 2 marks as ALT1	M1 A1 (8)
ALT 3	$2y=\ln(x+3)-1$ $e^{2y}=e^{\ln(x+3)-1}$	M1 M1
	$e^{2y}=\frac{x+3}{e}$ oe	M1
	$x-2=\frac{x+3}{e}$ oe	M1
	$x=\frac{3+2e}{e-1}=4.91$	dddM1
	Final 2 marks as main scheme	
There are potentially a large number of approaches to this question. Use the following general principles: Although the first 4 M marks are not dependent on each other, there must be sufficient work completed in the previous method mark to allow the step that is stated in the additional guidance to be completed. Look carefully at the relevant Practice item in OLS. Award full marks for correct values of *x* and *y*. If in any doubt or different methods are used not leading to correct answers, you **MUST** send this to review please.		
		Total 8 marks

Mark	Additional Guidance
M1	For correctly rearranging and taking logs to obtain $2y=\ln(x-2)$ or $y=\frac{1}{2}\ln(x-2)$
M1	For correct substitution of their expression for *y* or 2*y* into $\ln(x+3)-2y-1=0$
M1	For correct use of $\log_a \frac{x}{y}=\log_a x-\log_a y$ with their equation.
M1	For correctly converting their log equation to exponential form.
ddddM1	For correctly multiplying throughout to eliminate any denominators and collecting *x* terms on one side of the equation and making *x* subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $x=4.91$ or better e.g. exact form (Calc gives: 4.90988…) accept any answer which rounds to 4.91
M1	For correct substitution of their value for *x* into a correct equation leading to a value for *y*
A1	For $y=0.53$ Accept any answer which rounds 0.53 (calculator gives: 0.5340565…)
ALT 1	
M1	Correctly rearranging to make *x* the subject and correctly substituting into $\ln(x+3)-2y-1=0$
M1	A correct rearrangement of their equation in *y* to give an equation which then allows them to correctly convert their log equation to exponential form.
M1	Correctly applying the multiplication law for powers
M1	Correctly rearranging their equation and correctly factorising e^{2y}
ddddM1	For dividing by $e-1$ and converting to log form to make *y* the subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $y=0.53$ or better e.g. exact form (Calc gives: 0.5340565…), accept any answer which rounds to 0.53
M1	For correct substitution of *y* into a correct equation leading to a value for *x*
A1	For $x=4.91$ Accept any answer which rounds to 4.91 (Calc gives: 4.90988…)
ALT 2	
M1	Correctly rearranges and converts to exponential form to obtain $x=e^{2y+1}-3$
M1	A correct substitution of their *x* into $e^{2y}-x+2=0$
M1	Correct use of the multiplication law for powers
M1	Correct rearrangement of their equation and factorisation of e^{2y}
ddddM1	For dividing by $e-1$ and converting to log form to make *y* the subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $y=0.53$ or better e.g. exact form (Calc gives: 0.5340565…), accept any answer which rounds to 0.53
Final 2 marks	As ALT1
ALT 3	
M1	Correctly rearranges to make 2*y* the subject $2y=\ln(x+3)-1$
M1	Correctly raises each side to be a power of e $e^{2y}=e^{\ln(x+3)-1}$
M1	Correct use of the multiplication rule for powers and simplification to give $e^{2y}=\frac{x+3}{e}$
M1	Correct substitution of their expression for e^{2y} (usually equating the two equations)
ddddM1	For correctly multiplying throughout to eliminate any denominators and collecting *x* terms on one side of the equation and making *x* subject. Dependent on all previous method marks having been awarded and the rearrangement of their equation must be fully correct.
A1	For $x=4.91$ or better e.g. exact form (Calc gives: 4.90988…) accept any answer which rounds to 4.91
Final 2 marks	As main scheme

Question number	Scheme	Marks
10 a	$(y-1)+y^2=11 \Rightarrow y^2+y-12=0$ or $x^2+\left(x^2+1\right)^2=11 \Rightarrow x^4+3x^2-10=0$	M1
	$(y-3)(y+4)=0$ or $\left(x^2+5\right)\left(x^2-2\right)=0$	M1
	$(y=3\ \ y=-4)$ $\Rightarrow x^2=2\quad x^2=-5$	M1
	For stating the x coordinate of A is $-\sqrt{2}$ **and** the x coordinate of B is $\sqrt{2}$	A1 (4)
b	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)-(x^2+1)^2\ \mathrm{d}x$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2-(11-x^2)\mathrm{d}x$	M1
	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(-x^4-3x^2+10)\ \mathrm{d}x"$	A1ft
	$(\pi)\left[-\frac{x^5}{5}-x^3+10x\right]_{(-\sqrt{2})}^{(\sqrt{2})}$	M1
	$(\pi)\left[\left(-\frac{("\sqrt{2}")^5}{5}-("\sqrt{2}")^3+10"\sqrt{2}"\right)-\left(-\frac{("-\sqrt{2}")^5}{5}-("-\sqrt{2}")^3-10"\sqrt{2}"\right)\right]=63.98$	M1 A1 (5)
ALT		
	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)\ \mathrm{d}x-\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2\ \mathrm{d}x$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2\ \mathrm{d}x-\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)\ \mathrm{d}x$	{M1}
	$\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)\ \mathrm{d}x-\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^4+2x^2+1)\ \mathrm{d}x$	{A1ft}
	$(\pi)\left[11x-\frac{x^3}{3}\right]_{(-\sqrt{2})}^{(\sqrt{2})}-(\pi)\left[\frac{x^5}{5}+\frac{2x^3}{3}+x\right]_{(-\sqrt{2})}^{(\sqrt{2})}$	{M1}
	$(\pi)\left[\left(11("\sqrt{2}")-\frac{("\sqrt{2}")^3}{3}\right)-\left(11("-\sqrt{2}")-\frac{("-\sqrt{2}")^3}{3}\right)\right]$ $-(\pi)\left[\left(\frac{("\sqrt{2}")^5}{5}+\frac{2("\sqrt{2}")^3}{3}+"\sqrt{2}"\right)-\left(\frac{("-\sqrt{2}")^5}{5}+\frac{2("-\sqrt{2}")^3}{3}"-\sqrt{2}"\right)\right]$ $=63.98$	{M1} {A1} (5)
		Total 9 marks

Note: the candidate may also set up the integral as

$2\times\pi\int_0^{\sqrt{2}}(11-x^2)-(x^2+1)^2\ \mathrm{d}x$ or $2\times\pi\int_{-\sqrt{2}}^{0}(11-x^2)-(x^2+1)^2\ \mathrm{d}x$ or

$2\times\left(\pi\int_0^{\sqrt{2}}(11-x^2)\ \mathrm{d}x-\pi\int_0^{\sqrt{2}}(x^2+1)^2\ \mathrm{d}x\right)$ *or* $2\times\left(\pi\int_{-\sqrt{2}}^{0}(11-x^2)\ \mathrm{d}x-\pi\int_{-\sqrt{2}}^{0}(x^2+1)^2\ \mathrm{d}x\right)$

If it is clear that the integral(s) being used at any point needs to be multiplied by 2, all marks can be awarded, follow the same principles in the main or ALT schemes if the multiply by 2 is seen anywhere in the solution.
If multiply by 2 is nowhere present or implied by a final answer, the maximum mark is M0 A0 M1 M1 A0

Part	Mark	Additional Guidance
(a)	M1	For correctly rearranging to get $x^2 = y - 1$ and correctly substituting to get an equation of the form $ay^2 + by + c = 0 \quad a,b,c \neq 0$ **or** For correctly substituting $y = x^2 + 1$, an attempt to expand $(x^2+1)^2$ and attaining an equation of the form $dx^4 + ex^2 + f = 0 \quad d,e,f \neq 0$
	M1	For a complete attempt to solve either $ay^2 + by + c$ or $dx^4 + ex^2 + f$ by any valid method – see general guidance for definition of minimally acceptable attempt.
	M1	For correctly solving their quadratic equation in *y* or quartic equation in *x* **and** for obtaining a value(s) for x^2 or *x*. Award this mark if the candidate does not state or dismisses a solution where x^2 is negative.
	A1	For stating the *x* coordinate of *A* is $-\sqrt{2}$ **and** the *x* coordinate of *B* is $\sqrt{2}$ Decimal equivalent allowed.
(b)	M1	For $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2)-(x^2+1)^2 \text{ d}x$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2 - (11-x^2)\text{d}x$ Allow use of their limits from part a.
	A1ft	For expanding the bracket and simplifying to give $\pi\int_{-\sqrt{2}}^{\sqrt{2}}(-x^4-3x^2+10) \text{ d}x$ The ft is only for use of their limits from part a.
	M1	For correct integration of their expressions. There must be at least 3 terms overall and there must be a term in x^4 or y^4. It is not necessary for π or limits to be present for this mark.
	M1	For correct substitution of their limits seen into their changed expression. Each of the limits needs to be substituted into the expression correctly at least once. Allow a fully correct final answer to imply this mark. Brackets must be correct but can be recovered later.
	A1	For awrt 63.98 (Calc: 63.9775…) If a negative value is found and changed at the end to a positive value, this final A mark cannot be awarded.
ALT	M1	For $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2) \text{ d}x - \pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2 \text{ d}x$ or $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^2+1)^2 \text{ d}x - \pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2) \text{ d}x$ Allow use of their limits from part a.
	A1ft	For expanding the bracket and simplifying $\pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(11-x^2) \text{ d}x - \pi\int_{"-\sqrt{2}"}^{"\sqrt{2}"}(x^4+2x^2+1) \text{ d}x$ The ft is only for use of their limits from part a.
	M1	For correct integration of their expressions. There must be at least 3 terms overall and there must be a term in x^4 or y^4. It is not necessary for π or limits to be present for this mark.
	M1	For correct substitution of their limits seen into their changed expression. Each of the limits needs to be substituted into each expression correctly at least once. Allow a fully correct final answer, with correct integration shown, to imply this mark. Brackets must be correct but can be recovered later.
	A1	For awrt 63.98 (Calc: 63.9775…) If a negative value is found and changed at the end to a positive value, this final A mark cannot be awarded.

Question number	Scheme	Marks
11 a i	$\left(\overrightarrow{AN}=\right)\overrightarrow{AO}+\overrightarrow{ON}$ or $\left(\overrightarrow{AN}=\right)\overrightarrow{AO}+\frac{3}{4}\overrightarrow{OB}=-\mathbf{a}+\frac{3}{4}\mathbf{b}$	M1 A1
ii	$\left(\overrightarrow{BM}=\overrightarrow{BO}+\overrightarrow{OM}=\right)-\mathbf{b}+\frac{1}{2}\mathbf{a}$	B1 (A1 on ePen) (3)
b	$\left(\overrightarrow{AX}=\lambda\left(\pm\left(\text{their }\overrightarrow{AN}\right)\right)\right)=\lambda\left(\pm\text{"}-\mathbf{a}+\frac{3}{4}\mathbf{b}\text{"}\right)$	B1 ft
	$\left(\overrightarrow{AX}=\overrightarrow{AM}+\mu\left(\pm\left(\text{their }\overrightarrow{MB}\right)\right)=\right)-\frac{1}{2}\mathbf{a}+\mu\left(\pm\text{"}-\frac{1}{2}\mathbf{a}+\mathbf{b}\text{"}\right)$	M1
	Equating components gives $-\lambda=-\frac{1}{2}-\frac{1}{2}\mu$	M1
	and $\frac{3}{4}\lambda=\mu$	M1
	$-\lambda=-\frac{1}{2}-\frac{1}{2}\left(\frac{3}{4}\lambda\right)\Rightarrow\lambda=\frac{4}{5}$ *or* $\mu=\frac{3}{5}$	ddM1 A1
	So $AX:XN=4:1$	A1 (7)
ALT	$\left(\overrightarrow{AX}=\overrightarrow{AB}+\overrightarrow{BN}+\lambda\left(\pm\left(\text{their }\overrightarrow{AN}\right)\right)\right)=\mathbf{b}-\mathbf{a}-\frac{1}{4}\mathbf{b}+\lambda\left(\pm\text{"}-\mathbf{a}+\frac{3}{4}\mathbf{b}\text{"}\right)$	B1ft
	$\left(\overrightarrow{AX}=\overrightarrow{AM}+\mu\left(\pm\left(\text{their }\overrightarrow{MB}\right)\right)=\right)-\frac{1}{2}\mathbf{a}+\mu\left(\pm\text{"}-\frac{1}{2}\mathbf{a}+\mathbf{b}\text{"}\right)$	M1
	Equating components gives $-\frac{1}{2}-\frac{1}{2}\mu=-1+\lambda$	M1
	and $\mu=\frac{3}{4}-\frac{3}{4}\lambda$	M1
	$\mu=\frac{3}{4}-\frac{3}{4}\left(\frac{1}{2}-\frac{1}{2}\mu\right)\Rightarrow\lambda=\frac{1}{5}$ *or* $\mu=\frac{3}{5}$	ddM1 A1
	So $AX:XN=4:1$	A1 (7)

General principles for marking part b

B1 ft states a valid vector, with a parameter, that could be used in a solution to find the ratio required.

M1 states a second valid vector path, with a second distinct parameter that could be used to find the ratio required. **This must be a distinct route, different to that used already, not travelling along the same line.**

M1 This mark is obtained by correctly equating their components for **a. They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ**

M1 This mark is obtained by correctly equating their components for **b. They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ**

ddM1 For an attempt to solve their simultaneous equations. There can be errors, but it must be clear they are solving simultaneous equations to arrive at a value for λ or μ or their parameters.

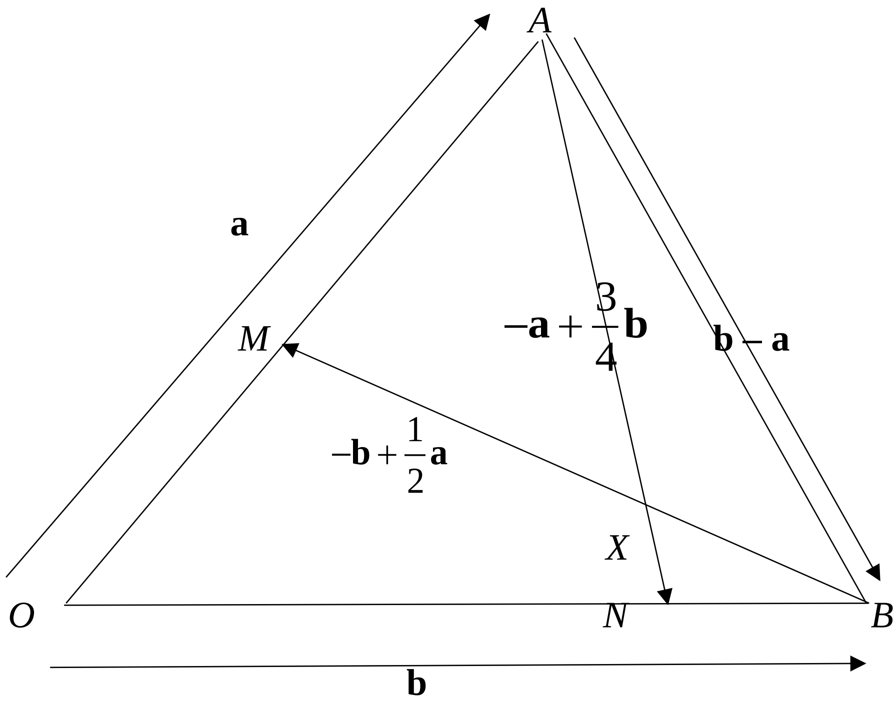
Dependent on both previous method marks.

A1 correct value for one of their parameters

A1 correct ratio

Part	Mark	Additional Guidance
(a) (i)	**M1**	For a correct vector path stated, a fully correct answer implies this mark.
	A1	For $-\mathbf{a}+\frac{3}{4}\mathbf{b}$
(ii)	**B1 (A1 ePen)**	For $-\mathbf{b}+\frac{1}{2}\mathbf{a}$
(b)	**B1 ft**	For $\overrightarrow{AX}=\lambda\ \pm$ their $\overrightarrow{AN}$ written in terms of vectors **a** and **b** $\left(\overrightarrow{AX}=\right)\lambda\left(\pm\left(\text{"}-\mathbf{a}+\frac{3}{4}\mathbf{b}\text{"}\right)\right)$ This may be seen embedded in working.
	M1	For e.g. $\overrightarrow{AX}=\overrightarrow{AM}+\mu\ \pm$ their $\overrightarrow{MB}$ written in terms of vectors **a** and **b** $\left(\overrightarrow{AX}=\right)-\frac{1}{2}\mathbf{a}+\mu\left(\pm\left(\text{"}-\frac{1}{2}\mathbf{a}+\mathbf{b}\text{"}\right)\right)$ This may be seen embedded in working. **This must be a distinct route, different to that used already, not travelling along the same line.**
	M1	For $-\lambda=-\frac{1}{2}-\frac{1}{2}\mu$ This mark is obtained by correctly equating their components for **a**, the example given is when their equations are set up using $\overrightarrow{AX}=\lambda\overrightarrow{AN}$ and $\overrightarrow{AX}=\overrightarrow{AM}+\mu\overrightarrow{MB}$ **They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.**
	M1	For $\frac{3}{4}\lambda=\mu$ This mark is obtained by correctly equating their components for **b**, the example given is when their equations are set up using $\overrightarrow{AX}=\lambda\overrightarrow{AN}$ and $\overrightarrow{AX}=\overrightarrow{AM}+\mu\overrightarrow{MB}$ **They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.**
	ddM1	For an attempt to solve their simultaneous equations. There can be errors, but it must be clear they are solving simultaneous equations to arrive at a value for λ or μ or their parameters. Dependent on both previous method marks.
	A1	There are a number of different correct answers here, depending on how they've set up their equations: For $\lambda=\frac{4}{5}$ or $\mu=\frac{3}{5}$ using $\overrightarrow{AX}=\lambda\overrightarrow{AN}$ and $\overrightarrow{AX}=\overrightarrow{AM}+\mu\overrightarrow{MB}$ For $\lambda=-\frac{4}{5}$ or $\mu=\frac{3}{5}$ using $\overrightarrow{AX}=\lambda\left(-\overrightarrow{AN}\right)$ and $\overrightarrow{AX}=\overrightarrow{AM}+\mu\overrightarrow{MB}$ For $\lambda=\frac{4}{5}$ or $\mu=-\frac{3}{5}$ using $\overrightarrow{AX}=\lambda\overrightarrow{AN}$ and $\overrightarrow{AX}=\overrightarrow{AM}+\mu\left(-\overrightarrow{MB}\right)$ For $\lambda=-\frac{4}{5}$ or $\mu=-\frac{3}{5}$ using $\overrightarrow{AX}=\lambda\left(-\overrightarrow{AN}\right)$ and $\overrightarrow{AX}=\overrightarrow{AM}+\mu\left(-\overrightarrow{MB}\right)$
	A1	For $AX:XN=4:1$

(b) ALT	**B1ft**	For $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BN} + \lambda\left(\pm \text{their } \overrightarrow{AN}\right)$ written in terms of **a** and **b** $\left(\overrightarrow{AX}\right) = \mathbf{b} - \mathbf{a} - \frac{1}{4}\mathbf{b} + \lambda\left(\pm\text{"}-\mathbf{a} + \frac{3}{4}\mathbf{b}\text{"}\right)$ This may be seen embedded in working.
	M1	For e.g. $\overrightarrow{AX} = \overrightarrow{AM} + \mu \pm \text{their } \overrightarrow{MB}$ written in terms of vectors **a** and **b** $\left(\overrightarrow{AX} = \right) - \frac{1}{2}\mathbf{a} + \mu\left(\pm\text{"}-\frac{1}{2}\mathbf{a} + \mathbf{b}\text{"}\right)$ This may be seen embedded in working. **This must be a distinct route, different to that used already, not travelling along the same line.**
	M1	For $-\frac{1}{2} - \frac{1}{2}\mu = -1 + \lambda$ This mark is obtained by correctly equating their components for **a**, the example given is when their equations are set up using $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BN} + \lambda\overrightarrow{AN}$ and $\overrightarrow{AX} = \overrightarrow{AM} + \mu\overrightarrow{MB}$ **They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.**
	M1	For $\mu = \frac{3}{4} - \frac{3}{4}\lambda$ This mark is obtained by correctly equating their components for **b**, the example given is when their equations are set up using $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BN} + \lambda\overrightarrow{AN}$ and $\overrightarrow{AX} = \overrightarrow{AM} + \mu\overrightarrow{MB}$ **They must have two relevant vectors, along two distinct paths and two distinct parameters. These do not need to be labelled λ and μ.**
	ddM1	For an attempt to solve their simultaneous equations. There can be errors, but it must be clear they are solving simultaneous equations to arrive at a value for λ or μ Dependent on both previous method marks.
	A1	There are a number of different correct answers here, depending on how they've set up their equations: For $\lambda = \frac{1}{5}$ or $\mu = \frac{3}{5}$ using $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BN} + \lambda\overrightarrow{AN}$ and $\overrightarrow{AX} = \overrightarrow{AM} + \mu\overrightarrow{MB}$ For $\lambda = -\frac{1}{5}$ or $\mu = \frac{3}{5}$ using $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BN} + \lambda\left(-\overrightarrow{AN}\right)$ and $\overrightarrow{AX} = \overrightarrow{AM} + \mu\overrightarrow{MB}$ For $\lambda = \frac{1}{5}$ or $\mu = -\frac{3}{5}$ using $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BN} + \lambda\overrightarrow{AN}$ and $\overrightarrow{AX} = \overrightarrow{AM} + \mu\left(-\overrightarrow{MB}\right)$ For $\lambda = -\frac{1}{5}$ or $\mu = -\frac{3}{5}$ using $\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BN} + \lambda\left(-\overrightarrow{AN}\right)$ and $\overrightarrow{AX} = \overrightarrow{AM} + \mu\left(-\overrightarrow{MB}\right)$
	A1	For $AX : XN = 4 : 1$


A
a
M
$-\mathbf{a}+\frac{3}{4}\mathbf{b}$
$\mathbf{b}-\mathbf{a}$
$-\mathbf{b}+\frac{1}{2}\mathbf{a}$
X
O
N
B
b

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