Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 02

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2020
Publications Code 4PM1_02_2001_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers may score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \text{ leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \text{ leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is <u>not</u> quoted, the method mark can be gained by implication from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

International GCSE Further Pure Mathematics – Paper 2 mark scheme

Question Number	Scheme	Marks
1	$\frac{ds}{dt} = 3t^2 + 8t - 27 = 8$	M1
	$3t^2 + 8t - 35\ (= 0)$	A1
	$(3t - 7)(t + 5) = 0$	
	$t = \frac{7}{3}$	M1A1cao
		[4]
M1	Attempt the differentiation and equate their result to 8. Power of at least one term to decrease and none to increase'	
A1	Obtain the correct 3TQ. Terms can be in any order and = 0 may be omitted.	
M1	Attempt to solve their 3TQ by any valid method. Must reach t = ...	
A1cao	For $t = \frac{7}{3}$ (negative answer must be omitted or eliminated) or t = 2.33 or better	

Question Number	Scheme	Marks
2(a)	$x \leqslant -1$	B1 (1)
(b)	$8x^2 + 10x - 3(< 0)$	
	$(4x-1)(2x+3)(< 0)$	M1
	$x = \frac{1}{4}$ $x = -\frac{3}{2}$	A1A1
	$-\frac{3}{2} < x < \frac{1}{4}$	A1ft (4)
(c)	$-\frac{3}{2} < x \leqslant -1$	B1 (1)
		[6]
(a)		
B1	For $x \leqslant -1$	
(b)	Accept decimals in (b) and (c)	
NB	The first 3 marks are for finding the critical values. Allow with < or = used.	
M1	Attempt to obtain the critical values by solving their 3TQ by any valid method.	
A1	Either CV correct	
A1	Second CV correct. Award these 2 marks if correct CVs seen in an inequality.	
A1ft	Inequality formed to indicate the values between their CVs. Must use < (Can be written in set language).	
NB	If CVs incorrect and only shown in the inequality, award 0/4 if no working shown for solving their 3TQ: if working shown M1A0A0A1 is available.	
(c)		
B1	For $-\frac{3}{2} < x \leqslant -1$ (no ft)	

Question Number	Scheme	Marks
3(a)	$AM=\sqrt{10^2-8^2}\ ,=6$	M1,A1 (2)
(b)	$\cos C=\frac{26^2+16^2-26^2}{2\times16\times26}=\frac{256}{832}\ \left(=\frac{4}{13}\ \text{ oe}\right)$	M1A1
	$\angle BCD=72°$	A1cao (3)
(c)	$AD=\sqrt{26^2-10^2}=24$ or $DM=\sqrt{26^2-8^2}=6\sqrt{17}$ oe (24.73...)	M1A1
	$\tan(\angle DMA)=\frac{24}{6}$ or $\cos(\angle DMA)=\frac{6}{6\sqrt{17}}$ or $\sin(\angle DMA)=\frac{24}{6\sqrt{17}}$	M1
	$\angle DMA=76°$	A1cao (4) **[9]**

(a)	
M1	Use Pythagoras, with a minus sign, to obtain the length of *AM*.
A1	Correct length obtained.
NB	Answers w/o working get both marks (use of (3,4,5) triangle)
(b)	
M1	Use the cosine rule in ΔBCD to obtain a numerical expression for cos *C*. Correct formula in either form may be quoted and substitution attempted or correct formula can be implied by the correct substitution. Complete method needed, so if another angle found first do not award this mark until a value for angle *BCD* is obtained.
A1	Correct value for the cosine obtained (Decimal to be awrt 0.308 (0.30769...)) Award by implication if final answer is awrt 72°.
A1cao	For 72° (from correct working)
ALT	**Use the isosceles triangle**
M1A1	$\cos C=\frac{8}{26}$ oe (Any trig function allowed provided work completed to a value for angle *BCD*)
A1cao	For 72° (from correct working)
(c)	
M1	Attempt the length of *AD* or *DM* using Pythagoras with a minus sign.
A1	Correct value for their choice of line,
M1	Use an appropriate trig function. The length of *AD* or *DM* must have been attempted with a + or a - sign.
A1cao	Correct answer.
	Penalise once only in (b) and (c) for failing to round as instructed.

Question Number	Scheme	Marks
4(a)	$\overrightarrow{DC} = (11\mathbf{i} - p\mathbf{j}) - (4\mathbf{i} - 2p\mathbf{j}) = 7\mathbf{i} + p\mathbf{j} = \overrightarrow{AB}$ OR: $\overrightarrow{BC} = (11\mathbf{i} - p\mathbf{j}) - (7\mathbf{i} + p\mathbf{j}) = 4\mathbf{i} - 2p\mathbf{j} = \overrightarrow{AD}$	M1A1
	Parallel and equal in length $\therefore$ Parallelogram	A1cso (3)
(b)	$\overrightarrow{BD} = (4\mathbf{i} - 2p\mathbf{j}) - (7\mathbf{i} + p\mathbf{j}) = -3\mathbf{i} - 3p\mathbf{j}$ (or $3(-\mathbf{i} - p\mathbf{j})$ oe	B1
	$\sqrt{9 + (3p)^2} = 3\sqrt{10}$ $(\Rightarrow 9 + 9p^2 = 90)$	M1
	$p = \pm 3$	A1 (3)
(c)	$(\pm)\frac{1}{3\sqrt{10}}(-3\mathbf{i} - 9\mathbf{j})$ oe	B1ft (1)
		[7]

	Accept column vectors throughout.
(a)	
M1	Attempt $\pm\overrightarrow{DC}$ or $\pm\overrightarrow{BC}$ using the difference of 2 appropriate vectors in component form.
A1	Show that $\pm\overrightarrow{DC}$ or $\pm\overrightarrow{BC} = \pm\overrightarrow{AB}$ or $\pm\overrightarrow{AD}$
A1cso	Suitable conclusion with reason from correct working. One pair of vectors only needed if reason is "parallel and equal". Both pairs needed if reason is "2 pairs of sides parallel/equal".
(b)	
B1	For a correct $\overrightarrow{BD}$ or $\overrightarrow{DB}$. No simplification needed.
M1	Use the given length of $\overrightarrow{BD}$ with the length of their $\overrightarrow{BD}$ to form an equation
A1	Obtain correct values for *p*. Both needed.
(c)	
B1ft	Use their positive value for *p* to obtain a unit vector (no simplification needed)

Question Number	Scheme	Marks
5		
(a)	$x^2-\frac{7}{2}x+2\ \ (=0)$	M1
	$2x^2-7x+4=0$	A1 (2)
(b)	$\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$	B1
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	M1
	$\frac{\frac{49}{4}-4}{2},=\frac{33}{8}$	dM1A1
	$x^2-\frac{33}{8}x+1\ \ (=0)$	M1
	$8x^2-33x+8=0$	A1 (6) **[8]**

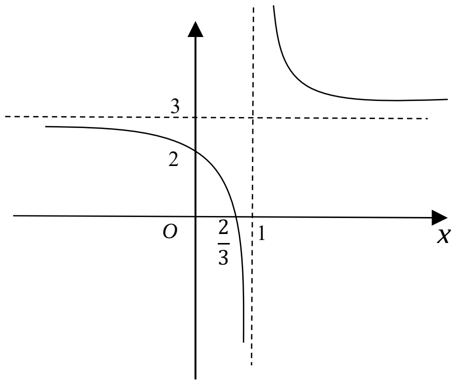
(a)	
M1	Use $x^2-(\text{sum of roots})x+\text{product of roots}$ (= 0 may be missing)
A1	Correct equation as shown or any **integer** multiple of this. Must have = 0 NB: A correct equation with no working scores 2
ALT	
M1	Eliminate α (or β) between the 2 equations and multiply through by α (or β)
A1	A correct quadratic equation with integer coefficients. Unknown can be α (or β)
NB;	isw any attempt to solve their equation.
(b)	
B1	Correct product of roots, seen explicitly or used.
M1	Attempt a single fraction for the sum of the roots with the numerator ready for substitution of known quantities. Denominator must be $\alpha\beta$.
dM1	Substitute numbers in their single fraction.
A1	Correct value for sum (as shown or equivalent fraction)
M1	Use $x^2-(\text{sum of roots})x+\text{product of roots}$ (= 0 may be missing)
A1	Correct equation as shown or any **integer** multiple of this. Must have = 0

Question Number	Scheme	Marks
6 (a)	When $x = \frac{3}{2}$ $y = 3 - \frac{9}{4} = \frac{3}{4}$ and $2y - \frac{3}{2} = 0$	M1
	So $\left(\frac{3}{2}, \frac{3}{4}\right)$ lies on both the line and the curve	A1cso (2)
(b)	$(\pi)\int_0^{\frac{3}{2}}(2x - x^2)^2\mathrm{d}x$	M1
	$= (\pi)\int_0^{\frac{3}{2}}(4x^2 + x^4 - 4x^3)\mathrm{d}x$	A1
	$= (\pi)\left[\frac{4x^3}{3} + \frac{x^5}{5} - x^4\right]_0^{\frac{3}{2}}$	dM1
	$= \frac{153(\pi)}{160}$	A1
	$\frac{153\pi}{160} - \frac{1}{3}\pi\left(\frac{3}{4}\right)^2\left(\frac{3}{2}\right) = \frac{27\pi}{40}$	ddM1A1cao (6)
		[8]
ALT:	$\pi\int_0^{\frac{3}{2}}\left((2x - x^2)^2 - \left(\frac{1}{2}x\right)^2\right)\mathrm{d}x$	M1
	$\pi\int_0^{\frac{3}{2}}(\frac{15}{4}x^2 - 4x^3 + x^4)\,\mathrm{d}x$	A1
	$\pi\left[\frac{5x^3}{4} - x^4 + \frac{x^5}{5}\right]_0^{\frac{3}{2}}$	dM1A1
	$\pi\left[\frac{135}{32} - \frac{81}{16} + \frac{243}{160}\right] = \frac{27\pi}{40}$	ddM1A1cao
(a)		
M1	Attempt to show that $\left(\frac{3}{2}, \frac{3}{4}\right)$ lies on the curve and the line. Any valid method including solving the equations allowed.	
A1cso	Appropriate conclusion following correct work. Verification, as shown, needs a conclusion. Solving the equations to obtain $\frac{x}{2} = 2x - x^2$ or $y = 4y - 4y^2$ and hence coordinates of A needs no conclusion. M1 for reaching coords, A1 for correct coords (decimals allowed)	

Question Number	Scheme	Marks
(b)	**Algebraic integration must be seen – otherwise no marks.** The first 4 marks can be awarded with or without π provided the work is consistent. The first 3 marks can be awarded if no limits are shown.	
M1	Correct integral, with or without π. Limits may be missing – ignore any shown.	
A1	Square the bracket correctly.	
dM1	Attempt the integration of their integrand. The power of at least one term should increase and no power should decrease. Ignore limits.	
A1	Substitute the correct limits and obtain $\frac{153}{160}$ or $\frac{153\pi}{160}$ (0.95625(pi))	
ddM1	Subtract the volume of the cone from their previous answer. Both terms to include π	
A1cao	Correct final answer (0.675pi)	
ALT:	**See above for general instructions re integration**	
M1	Integral must be the difference of 2 squared terms	
A1	Correct integrand after squaring, need not be simplified	
dM1	Attempt the integration of their integrand. The power of at least one term should increase and no power should decrease.	
A1	Correct result	
ddM1	Substitute their limits	
A1cao	Correct final answer.	

Question Number	Scheme	Marks
7(a)	$\dfrac{ar^7}{ar^6}=\dfrac{1152}{192}\quad (=6)=r$	B1
	4th term $=\dfrac{192}{6^3}$ or $\dfrac{1152}{6^4}\quad =\dfrac{8}{9}$	M1A1 (3)
(b)	$\dfrac{t_3}{r}+t_3+rt_3 \implies \dfrac{24}{r}+24+24r=-36$	M1A1 NB B1B1 on e-PEN
	$24+24r+24r^2=-36r$	dM1
	$24r^2+60r+24=2r^2+5r+2=0$ *	ddM1A1cso (5)
(c)	$2r^2+5r+2=0 \Rightarrow (2r+1)(r+2)=0 \Rightarrow r=-\dfrac{1}{2}$	M1A1
	$S=\dfrac{a}{1-r}=\dfrac{24\div\left(-\frac{1}{2}\right)^2}{1-\left(-\frac{1}{2}\right)},\ =64$	M1,A1 (4)
		[12]
(a)		
B1	Obtain a correct value for *r*. Fraction need not be simplified	
M1	Use their *r* and either the 7th or 8th term divided by the appropriate power of *r* to obtain the 4th term as a fraction – no need to simplify	
A1	$\dfrac{8}{9}$	
ALT	M1 Find *a* (=1/243) and use ar^3 A1 Correct answer	
(b)		
M1	Use the given information to obtain an equation in *r*	
A1	Correct equation	
dM1	Eliminate the fraction	
ddM1	Obtain a 3TQ, terms in any order	
A1cso	Reach the given result with no errors in the working	
(c)		
M1	Solve the given quadratic by any valid method. Must reach a value of *r*	
A1	Correct value of *r* (Ignore second answer if given)	
M1	Use the formula for the sum to infinity with their *r* **provided** $\lvert r\rvert<1$. *a* must be 24 divided by (their $r)^2$	
A1	Correct answer.	

Question Number	Scheme	Marks
8	$y = e^{3x}\sin 2x \quad \frac{dy}{dx} = 2e^{3x}\cos 2x + 3e^{3x}\sin 2x$	M1A1
	$\frac{d^2y}{dx^2} = \left(-4e^{3x}\sin 2x + 6e^{3x}\cos 2x\right) + \left(6e^{3x}\cos 2x + 9e^{3x}\sin 2x\right)$	M1A1A1
	$= 12e^{3x}\cos 2x + 5e^{3x}\sin 2x$	
	$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y$	
	$= 12e^{3x}\cos 2x + 5e^{3x}\sin 2x - 6\left(2e^{3x}\cos 2x + 3e^{3x}\sin 2x\right) + 13e^{3x}\sin 2x$	dM1
	$= 12e^{3x}\cos 2x + 5e^{3x}\sin 2x - 12e^{3x}\cos 2x - 18e^{3x}\sin 2x + 13e^{3x}\sin 2x$	ddM1
	$= 0$ *	A1cso **[8]**
ALT	$\frac{dy}{dx} = 2e^{3x}\cos 2x + 3e^{3x}\sin 2x$	M1A1
	$\frac{dy}{dx} = 2e^{3x}\cos 2x + 3y$	
	$\frac{d^2y}{dx^2} = \left(-4e^{3x}\sin 2x + 6e^{3x}\cos 2x\right) + 3\frac{dy}{dx}$	M1A1A1
	$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = \left(-4e^{3x}\sin 2x + 6e^{3x}\cos 2x + 3\frac{dy}{dx}\right) - 6\frac{dy}{dx} + 13y$	dM1
	$= -13y + 6e^{3x}\cos 2x + 9e^{3x}\sin 2x - 3\frac{dy}{dx} + 13y$	
	$= -13y + 3\frac{dy}{dx} - 3\frac{dy}{dx} + 13y = 0$ *	ddM1A1cso **[8]**
M1	Attempt the product rule. 2 terms of the form $\pm ke^{3x}\cos 2x$ and $\pm le^{3x}\sin 2x$ with $k = 1$ or 2 and $l = 1$ or 3	
A1	Fully correct first derivative	
M1	Attempt the second derivative using the product rule *correctly* on either term. Must have at least one of the terms in the first derivative fully correct.	
A1 A1	A1 for each fully correct bracket	
dM1	Substitute their derivatives and y in $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y$ Depends on both previous M marks This and the following M mark may be awarded together.	
ddM1	Remove the brackets	
A1cso	Reach "0" from fully correct work.	
ALT M1A1	As above	
M1	Replace sin term with a y term and attempt the second derivative using the product rule on first term.	
A1A1	A1 Correct bracket A1 Correct second term	
dM1	As above	
ddM1	Obtain an expression which is either all derivatives plus y terms or all trig terms	
A1cso	Reach "0" from fully correct work	

Question Number	Scheme	Marks
9(a)(i)	$\frac{2}{p} = 2$ so $p = 1$*	M1A1cso
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{q(x-1)-(qx-2)}{(x-1)^2}$	M1A1
	When $x = 0$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-q+2}{1} = -1, \Rightarrow q = 3$	M1A1,A1 (7)
(b)		B1ft B1ft B1 B1ft B1ft (5)
(c)	$x + 2 = \frac{3x-2}{x-1}$	M1
	$x^2 - 2x = 0$	M1
	$x(x-2) = 0$ $x = 2$	dM1A1cao(4) **[16]**

(a)(i)M1	Set $x = 0$ in the curve equation and equate result to 2. Obtain a value for p.
A1cso	Correct value of p obtained from a correct equation.
(ii)M1	Attempt the quotient rule. (formula is given on formula page). Denominator must be $(x-1)^2$. Numerator to be $q(x-1)-(qx-2)$ or $(qx-2)-q(x-1)$ **Must use $p = 1$ now or later.**
A1	Fully correct derivative
ALT	Use product rule: $\frac{\mathrm{d}y}{\mathrm{d}x} = q(x-1)^{-1} - (qx-2)(x-1)^{-2}$ M1 for attempt with 2 terms similar to above, either term to be correct A1 Both terms correct
M1	Set $x = 0$ in their derivative and equate to -1
A1	Correct equation
A1	$q = 3$

Question Number	Scheme	Marks
(b)	**No value for *q***: B0B1B0B1B1 available. **Incorrect *q*:** B1B1B0B1B1 available.	
B1ft	Equations of asymptotes seen or lines parallel to axes passing through $x = 1$, $y = 3$ drawn. $y = 3$ or their q. **Must have a value for *q*.**	
B1ft	Coordinates of crossing points seen explicitly or marked on the sketch. Must have $y = 2$; may have $x = 2/q$ (value for q not needed)	
B1	Two branches in the correct "quadrants" Must have $q = 3$ for this mark.	
	Asymptotes drawn.	
B1ft	There must be at least one branch of the curve drawn and 2 asymptotes drawn and labelled on the diagram by showing the coords of the points where they cross the axes or with their equations.	
	The curve must not touch (or cross) either asymptote. ft their asymptotes**, inc $y = q$**	
B1ft	Both crossing points clearly marked on their diagram. ft their crossing points .	
(c)		
M1	Eliminate y between the line and the curve equation. May use q or their value for q	
M1	Obtain a 2 or 3 term quadratic. May use q or their value for q.	
dM1	Solve their equation to obtain 1 or 2 values of x Depends on both M marks above.	
A1cao	$x = 2$ from a correct equation. If $x = 0$ is seen it must be clear that $x = 2$ is the only answer	
	If x is eliminated: M1 elimination M1 obtain quadratic in y M1 solve for y	
	A1 complete to a single value of x	

Question Number	Scheme	Marks
10	$\frac{\mathrm{d}V}{\mathrm{d}t} = 40 \text{ (cm}^3/\text{s)}$	B1
	$A = 4\pi r^2 \quad \frac{\mathrm{d}A}{\mathrm{d}r} = 8\pi r$	M1A1
	$V = \frac{4}{3}\pi r^3 \quad \frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	M1A1
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}, = 8\pi r \times \frac{1}{4\pi r^2} \times 40 \quad (= \frac{80}{r})$	M1,A1ft
	$r = 4$ so $\frac{80}{4} = 20 \text{ (cm}^2/\text{s)}$	dM1A1cao **[9]**
	Any letters can be used for volume and area, inc *SA* for area, but their choice must be used consistently.	
B1	State or use $\frac{\mathrm{d}V}{\mathrm{d}t} = 40 \text{ (cm}^3/\text{s)}$ (units not needed)	
M1	Attempt to differentiate $4\pi r^2$ with respect to r (Formula for area of sphere is given on formula page)	
A1	Correct derivative = d*A*/d*t*	
M1	Attempt to differentiate $\frac{4}{3}\pi r^3$ with respect to r (Formula for volume of sphere is given on formula page)	
A1	Correct derivative = d*A*/d*t*	
M1	Show (or use) a useful chain rule. Terms can be in any order as long as it is possible to obtain d*A*/d*t* from it. OR Use chain rule twice to obtain an expression from which d*A*/d*t* could be obtained.	
A1ft	Substitute their expressions for the 3 derivatives in their chain rule. Need not be simplified.	
dM1	Use the resulting expression(s) with $r = 4$ to obtain a value for d*A*/d*t* All previous M marks needed.	
A1cao +cso	Correct value, units may be missing. Solution must be correct.	

Question Number	Scheme	Marks
11(a)	$(3\sin A\cos B - 3\cos A\sin B) = (\sin A\cos B + \cos A\sin B)$	M1
	$2\sin A\cos B = 4\cos A\sin B$	M1
	$\rightarrow \dfrac{\sin A}{\cos A} = 2\dfrac{\sin B}{\cos B}$	M1
	$\tan A = 2\tan B \quad k = 2$	A1 (4)
(b)	$\dfrac{(\cos^4\theta - \sin^4\theta)}{\cos^2\theta} = \dfrac{(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)}{\cos^2\theta}$	M1
	$= \dfrac{(\cos^2\theta - \sin^2\theta)}{\cos^2\theta}$	M1
	$= 1 - \tan^2\theta$ *	A1 cso (3)
ALT 1	$1-\tan^2\theta = 1-\dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}$	M1
	$= \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\times\left(\cos^2\theta+\sin^2\theta\right)$	M1
	$= \dfrac{\cos^4\theta-\sin^4\theta}{\cos^2\theta}$	A1
ALT 2	$\dfrac{\cos^4\theta-\sin^4\theta}{\cos^2\theta} = \cos^2\theta - \dfrac{\sin^4\theta}{\cos^2\theta} = \cos^2\theta - \tan^2\theta\sin^2\theta$	M1 Eliminate 4th powers
	$= \cos^2\theta - \tan^2\theta\left(1-\cos^2\theta\right)$	M1 Eliminate $\sin^2$
	$= \cos^2\theta - \tan^2\theta + \sin^2\theta = 1-\tan^2\theta$	A1
(c)(i)	$\cos(45-30)$ or $\cos(60-45) = \dfrac{1}{2}\times\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{3}}{2}\times\dfrac{\sqrt{2}}{2}$	M1
	$= \dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{6}}{4} = \dfrac{\sqrt{2}+\sqrt{6}}{4}$ *	A1cso (2)
ALT	By using double angle formula: $\cos^2 15° = \dfrac{1}{2}\left(1+\cos 30°\right) = \dfrac{1}{2}\left(1+\dfrac{\sqrt{3}}{2}\right)$	M1
	Leading to the *given* answer. $\cos 15° = \sqrt{\left(\dfrac{2+\sqrt{3}}{4}\right)}$ or $\dfrac{\sqrt{2+\sqrt{3}}}{2}$ must be seen.	A1

Question Number	Scheme	Marks
(ii)	$\tan 255 = \tan 75$	B1
	$= \tan(30+45) = \dfrac{\tan 30 + \tan 45}{1 - \tan 30 \tan 45}$	M1
	$= \dfrac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}}$	dM1
	$\dfrac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \dfrac{3+\sqrt{3}}{3-\sqrt{3}}$ *	A1cso (4)
		[13]

(a)	
M1	Expand both sides of the equation using correct formulae
M1	Collect like terms from their expansions. (Not dependent)
M1	Divide through by $\cos A \cos B$
A1	Replace each fraction with the appropriate tangent and show $k = 2$ (value need not be shown explicitly)
(b)	
M1	Factorise the numerator using the difference of 2 squares.
M1	Replace $\sin^2\theta + \cos^2\theta$ with 1
A1cso	Divide both terms by $\cos^2\theta$ and obtain the *given* answer with no errors seen.
ALT 1	
M1	Use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and obtain a single fraction with no tan
M1	Indicate multiplication by $\sin^2\theta + \cos^2\theta$
A1cso	Multiply and obtain the *given* answer with no errors seen.
(c)(i)	
M1	Express 15 as the difference of 2 suitable numbers, expand using a correct formula and substitute the correct exact values for the trig functions (substitution must be shown).
A1cso	Simplify and combine the fractions to obtain the *given* answer with no errors seen.
(ii)	
B1	$\tan 255 = \tan 75$ seen explicitly or used. OR eg tan(210 + 45) – give B1 for tan 210 = tan 30 used
M1	Express 75 as 30 + 45 and expand $\tan(30+45)$ using the correct formula (given on the formula page) OR expand tan (210 + 45) If 75 = 15 + 60 is used tan 15 can be obtained from a calculator but must be in exact form..
dM1	Substitute the correct exact values for the trig functions
A1cso	Simplify the fractions to obtain the *given* answer with full working and no errors seen.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom