Please check the examination details be	elow before entering your candidate information
Candidate surname	Other names
Centre Number Candidate N	lumber
<b>Pearson Edexcel Inter</b>	national GCSE
Time 2 hours	Paper reference 4PM1/01R
<b>Further Pure Mat</b>	
PAPER 1R	
Calculators may be used.	Total Marks

### **Instructions**

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- You must NOT write anything on the formulae page.
   Anything you write on the formulae page will gain NO credit.

### **Information**

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

### **Advice**

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶





### **International GCSE in Further Pure Mathematics Formulae sheet**

### Mensuration

**Surface area of sphere** =  $4\pi r^2$ 

Curved surface area of cone =  $\pi r \times \text{slant height}$ 

Volume of sphere =  $\frac{4}{3}\pi r^3$ 

### **Series**

### **Arithmetic series**

Sum to *n* terms,  $S_n = \frac{n}{2} [2a + (n-1)d]$ 

### **Geometric series**

Sum to *n* terms, 
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity, 
$$S_{\infty} = \frac{a}{1-r} |r| < 1$$

### **Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for  $|x| < 1, n \in \mathbb{Q}$ 

### **Calculus**

### **Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x} \left( \frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

### **Trigonometry**

### Cosine rule

In triangle *ABC*:  $a^2 = b^2 + c^2 - 2bc \cos A$ 

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

### Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



### Answer all ELEVEN questions.

### Write your answers in the spaces provided.

### You must write down all the stages in your working.

(a) On the grid below, draw the graph of the line with equation

(i) 
$$4x + 5y = 20$$

(i) 
$$4x + 5y = 20$$
 (ii)  $3y - 4x = -12$ 

**(2)** 

(b) Show, by shading on the grid, the region R defined by the inequalities

$$4x + 5y \leq 20$$

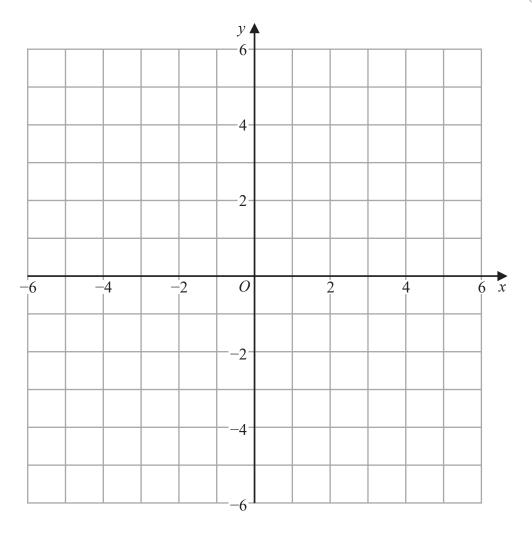
$$4x + 5y \leqslant 20$$
  $3y - 4x \geqslant -12$   $y \leqslant 3$   $x \geqslant 1$ 

$$v \leq 3$$

$$x \geqslant 1$$

Label the region *R*.

**(2)** 



Turn over for a spare grid if you need to redraw your graph.



Question 1 continued		



# **Question 1 continued** Only use this grid if you need to redraw your graph. 2 \_2 -6 -40 -2



(Total for Question 1 is 4 marks)

2	The <i>n</i> th term of an arithmetic series is $u_n$	
	Given that $u_5 = 46$ and that $u_{20} = 181$	
	(a) find	
	(i) the common difference of this series,	
	(ii) the first term of this series.	(4)
	50	(4)
	(b) Evaluate $\sum_{n=21}^{50} u_n$	
	n=21	(4)





3	The point $A$ has coordinates $(1, 7)$ and the point $B$ has coordinates $(9, 3)$ The line $I$ is the perpendicular bisector of $AB$	
	(a) Find an equation of $l$	(5)
	The line <i>l</i> crosses the <i>x</i> -axis at the point <i>C</i> (b) Find the area of the triangle <i>ABC</i>	
	(b) Find the area of the triangle ABC	(5)





O  $\frac{\pi}{3}$  rad R B

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows sector OAB of a circle with centre O and radius r cm.

The angle  $AOB = \frac{\pi}{3}$  radians.

The point D divides OA in the ratio 3:1

The area of the region R, shown shaded in Figure 1, is  $79.5 \,\mathrm{cm}^2$ 

(a) Calculate the value, to 2 significant figures, of r

(4)

(b) Calculate the perimeter, in cm to 2 significant figures, of the region R

(4)



(a) Write down the expansion of f(x) in ascending powers of x up to and including the term in  $x^3$ 

Give each coefficient in terms of n and a

**(2)** 

In the expansion of f(x), the coefficient of x is 8 and the coefficient of  $x^2$  is 30

(b) Find the value of n and the value of a

**(4)** 

(c) Calculate the coefficient of  $x^3$  in f(x)

(2)

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6	The points $P$ ,	O $B$	and Car	e the ve	ertices of	a anadrilateral	$P \cap R \subseteq$	such	that
O	The points P,	Q, T	t and 5 ar	e me ve	cruces of a	a quadriiaterai	PUKS	Such	mai

$$\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j}$$

$$\overrightarrow{PR}$$
 -i + 18j

$$\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j}$$
  $\overrightarrow{PR} - \mathbf{i} + 18\mathbf{j}$   $\overrightarrow{PS} = -3\mathbf{i} + 15\mathbf{j}$ 

(a) Show that *PQRS* is a parallelogram.

**(4)** 

(b) Find a unit vector parallel to  $\overrightarrow{QS}$  as a simplified expression in terms of **i** and **j** 

**(4)** 

The point *T* lies on *QS* such that QT: TS = 5:8

(c) Find  $\overrightarrow{PT}$  as a simplified expression in terms of **i** and **j** 

(2)




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Question 6 continued	



y cm 5x cm 4x cm

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length 3x cm, 4x cm and 5x cm.

The length of the prism is y cm.

The total surface area of the five faces of the prism is 144 cm<sup>2</sup>

The volume of the prism is  $V \text{cm}^3$ 

(a) Show that

$$V = 72x - 6x^3$$

(5)

Given that x can vary,

(b) use calculus to find the value of x for which V is a maximum, justifying that this value gives a maximum value of V

**(4)** 

(c) Find the maximum value of V

**(2)** 



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Question 7 continued		





8	The curve C has equation $y = 2x^2 - \sin x$	
	The point $A$ on $C$ has $x$ coordinate $\pi$	
	Show that an equation of the normal to $C$ at the point $A$ is	
	$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0$	(0)
		(8)





(a) Complete the table of values for

$$y = \frac{x^3 + 4}{5 - x}$$

giving your answers to 2 decimal places.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-0.57	0.10	0.5		0.8		1.25		4

(2)

(b) On the grid opposite, draw the graph of

$$y = \frac{x^3 + 4}{5 - x} \qquad \text{for } -2 \leqslant x \leqslant 2$$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation  $x^3 - x^2 + 8x - 11 = 0$  in the interval  $-2 \le x \le 2$ 

**(5)** 

## Question 9 continued 3.5 3 2.5 1.5 0.5 2x0.5 -0.5



Turn over for a spare grid if you need to redraw your graph.

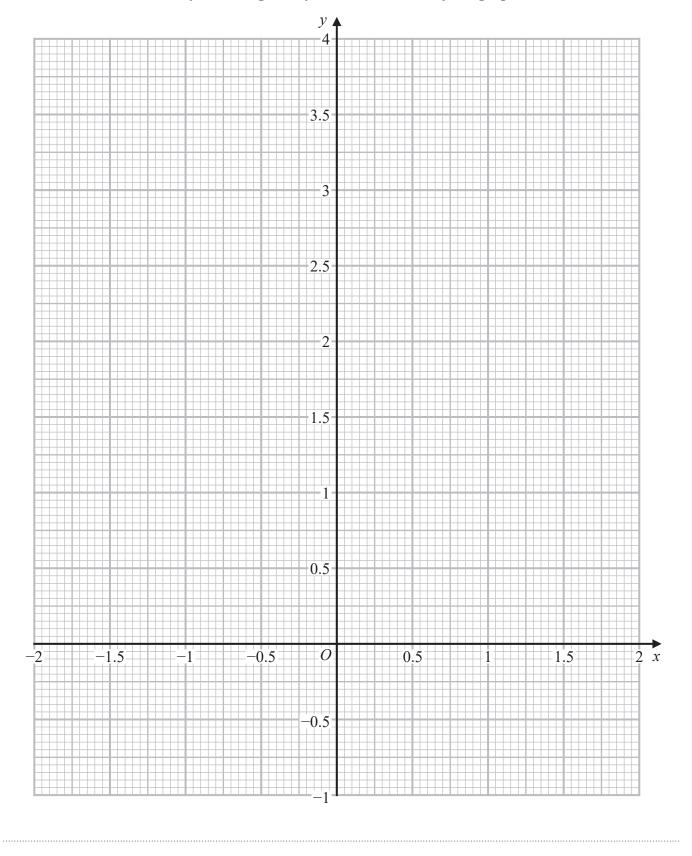
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Question 9 continued	



## Question 9 continued

Only use this grid if you need to redraw your graph.



(Total for Question 9 is 9 marks)



10 (a) Use the factor theorem to show that (4x - 3) is a factor of

$$16x^3 + 11x - 15$$

(2)

- (b) Using formulae given on page 2, show that
  - (i)  $\sin 2\theta = 2\sin\theta\cos\theta$
  - (ii)  $\cos 2\theta = 2\cos^2 \theta 1$

**(5)** 

(c) Show that the equation

$$27\cos\theta\cos2\theta + 19\sin\theta\sin2\theta - 15 = 0$$

becomes the equation

$$16x^3 + 11x - 15 = 0$$

by using the substitution  $x = \cos \theta$ 

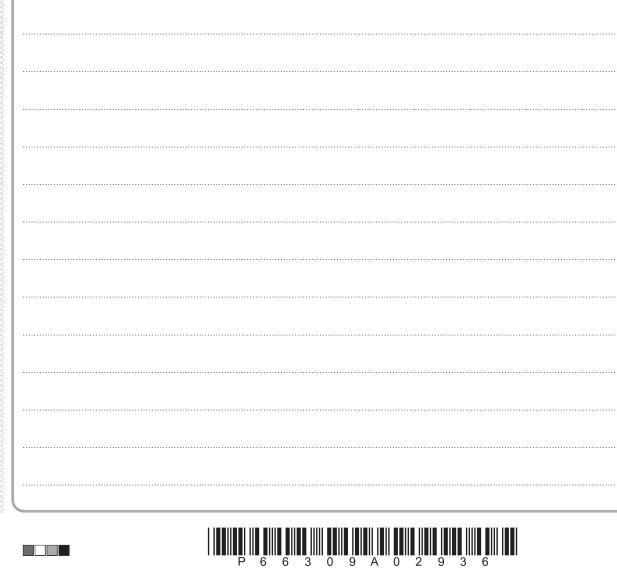
**(4)** 

(d) Hence show that any solution of the equation

$$27\cos\theta\cos2\theta + 19\sin\theta\sin2\theta - 15 = 0$$

is given by  $\cos \theta = \frac{3}{4}$ 

**(4)** 

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Question 10 continued		



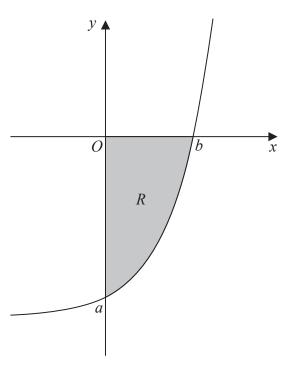


Diagram **NOT** accurately drawn

Figure 3

The finite region R, shown shaded in Figure 3, is bounded by the curve with equation  $y = e^{2x} - 9$  and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates (0, a) and (b, 0)

- (a) (i) Find the value of a
  - (ii) Show that  $b = \ln 3$

(3)

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the volume of the solid generated.

Give your answer in the form  $\pi(p \ln 3 + q)$ , where p and q are integers.

**(6)** 



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Question 11 continued	





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Question 11 continued		
	(Total for Question 11 is 9 marks)	
	TOTAL FOR PAPER IS 100 MARKS	

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